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Onset of Bénard–Marangoni convection in a micropolar fluid

Z. Alloui [a,*], P. Vasseur [a,b]

[a] École Polytechnique, Université de Montréal, C.P. 6079, Succ. «Centre Ville», Montréal, Québec, Canada H3C 3A7

[b] Laboratoire des Technologies Innovantes, Université Jules Vernes d'Amiens, rue des Facultés le Bailly, 800025 Amiens Cedex, France

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ABSTRACT

This investigation reports on a stability analysis of the quiescent state within a horizontal layer of a micropolar fluid. The horizontal boundaries are considered rigid–rigid, rigid–free or free–free. Thermal boundary conditions of the Neumann type are applied on the boundaries of the system. The critical Rayleigh and Marangoni numbers for the onset of supercritical convection of micropolar are predicted analytically on the basis of the parallel flow approximation. The onset of motion is found to depend on the materials parameters K, B, λ and the micro-rotation boundary condition n. Furthermore, a linear stability analysis is conducted yielding numerically the critical Rayleigh and Marangoni numbers for the onset of motion from the rest state. A good agreement is observed between the analytical model and the numerical simulations.

1. Introduction

The micropolar fluid theory, which deals with fluids in which gyrational effects are important, was developed by Eringen [1–4] in order to describe some physical systems which do not satisfy the Navier–Stokes model. Micropolar fluids, which posses inertia not only for translation but also for rotation, are able to describe the behavior of colloidal solutions, suspension solutions, the extrusion of polymer fluids, continuous casting glass–fiber, animal bloods, cooling of a metallic plate in a bath production, paper production and metal extraction.

The first study concerning thermal instability of a micropolar fluid layer heated from below, with free upper and lower boundaries, seems to be due to Ahmadi [5]. The existence of cellular convection at the onset of convection was demonstrated by this author. Rama Rao [6] studied the onset of convection of a heat conducting micropolar fluid layer confined between two horizontal rigid boundaries. The heat induced by microrotation leads to the onset of instability not only due to adverse temperature gradients but also for positive ones. The possibility of overstable motions of micropolar fluids heated from below has been investigated by Pérez-Garcia and Rubi [7]. It was found that such motions are possible only for fluids with a very large coupling parameter between the spin flux and the heat flux. The instability of rotating micropolar fluids has been investigated by Sastry and Ramamohan [8]. The effects of a magnetic field and nonuniform temperature profiles on Marangoni convection in micropolar fluids confined between a lower rigid isothermal boundary and an upper free, constant heat flux, boundary was investigated by Rudraiah et al. [9]. It was demonstrated by these authors that micropolar fluids heated from below are more stable when compare to the pure viscous fluid situation. Siddheshwar and Pranesh [10] investigated the influence of suction-injection on the linear stability of Rayleigh–Benard Marangoni convection in a horizontal layer with an upper free boundary. The micropolar fluid layer was found to be more stable than the classical pure fluid layer. The effects of through flow and magnetic field on the onset of Benard convection in a horizontal layer of micropolar fluid confined between two rigid, isothermal and micro-rotation free, boundaries have been studied by Narasimha Murty [11]. The critical Rayleigh number was predicted on the basis of a single-term Galerkin technique. Rayleigh–Bénard convection in a micropolar ferromagnetic fluid has been investigated analytically by Abraham [12] for a layer with free–free, isothermal, spin-vanishing magnetic boundaries. It was demonstrated that the micropolar ferromagnetic fluid layer heated from below is more stable as compared to the Newtonian ferromagnetic fluid. Narasimha Murty [13] investigated the effects of through flow and uniform magnetic field on the onset of Marangoni convection in micropolar fluids. Sharma and Gupta [14] performed a study of the stability of thermal convection in micropolar fluids under the influence of a uniform vertical rotation field. It was reported that Rayleigh number for the case of overstability and stationary convection increases with increase of rotation parameters and decreases with increase of micropolar coefficients. The effect of a non-uniform basic temperature gradient on the onset of Marangoni convection in a horizontal micropolar fluid layer was considered by Melviana et al. [15]. It was assumed that the layer is bounded below by a rigid plate and above by a non-deformable

* Corresponding author. Tel.: +1 514 340 4711x3619; fax: +1 514 340 5917.
E-mail address: zineddine.alloui@polymtl.ca (Z. Alloui).
URL: http://www.meca.polymtl.ca/convection (Z. Alloui).

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Nomenclature

A	aspect ratio of the cavity, L'/H'
A_c	wave length of the convection cell, π/k_c
B	micro-inertia parameter, H'^2/j
C	constant temperature gradient in x-direction
g	gravitational acceleration
H'	height of fluid layer
j	micro-inertia per unit mass
k	thermal conductivity
k_c	wave number
K	vortex viscosity parameter, κ/μ
L'	width of fluid layer
Ma	Marangoni number, $\sigma_0\gamma_T\Delta T'H'/\alpha\mu$
Ma_c	critical Marangoni number, Eq. (41)
Nu	Nusselt number, Eq. (7)
N	dimensionless angular velocity, $N'H'^2/\alpha$
n	dimensionless micro-gyration parameter equation (4)
Pr	Prandtl number, ν/α
q'	constant heat flux per unit area
Ra	thermal Rayleigh number, $g\beta'_T\Delta T'H'^3/\alpha\nu$
Ra_c	critical Rayleigh number, Eq. (24)
t	dimensionless time, $t'\alpha/H'^2$
T	dimensionless temperature, $(T'-T'_0)/\Delta T'$
u	dimensionless velocity in x-direction, $u'H'/\alpha$
v	dimensionless velocity in y-direction, $v'H'/\alpha$
x	dimensionless coordinate axis, x'/H'
y	dimensionless coordinate axis, y'/H'

Greek symbols

α	thermal diffusivity
β'_T	thermal expansion coefficient
γ_T	thermal surface tension gradient
μ	dynamic viscosity
ν	kinematic viscosity of fluid, μ/ρ
λ	material parameter, $\gamma/(\mu j)$
ρ	density of fluid
σ	fluid surface tension
Ψ	dimensionless stream function, Ψ'/α
γ	spin gradient viscosity
κ	vortex viscosity

Subscript

0	reference state
c	refers to critical conditions

Superscript

$'$	refers to dimensional variable

free surface subjected to a constant heat flux. At these boundaries the microrotation is assumed to be vanished. The influence of various parameters on the onset of convection is discussed. The effect of a magnetic field on the onset of Marangoni convection in micropolar fluid has been considered by Mahmud et al. [16]. The presence of the magnetic field was found to always have a stability effect of increasing the critical Marangoni number. A linear stability analysis was performed by Idris et al. [17] to study the effect of non-uniform basic temperature gradients on the onset of Bénard–Marangoni convection in a micropolar fluid. The influence of various parameters on the onset of convection has been analyzed by these authors. It was found that the presence of micron-sized suspended particles delays the onset of convection. Recently, an analytical and numerical study of natural convection in a shallow cavity filled with a micropolar fluid has been reported by Alloui and Vasseur [18]. Among other results, the critical Rayleigh for the onset of convection was obtained in a close form in terms of the governing parameters of the problem.

The aim of the present paper is to study the effects of various hydrodynamic boundary conditions such as free–free, rigid–free on the onset of Bénard–Maragoni convection of a horizontal layer of micropolar fluid. The critical conditions for the onset of convection shall be determined by both analytical and numerical methods. The paper is organized as follows. In the next sections, the formulation of the problem is presented. A linear stability analysis is conducted to predict numerically the critical Rayleigh and Marangoni numbers for the onset of motion from the rest state. An approximate analytical solution, based on the parallel flow approximation, is then proposed. The last section contains some concluding remarks.

2. Mathematical formulation

The configuration considered in this study is a horizontal shallow cavity, of thickness H' and width L' filled with a micropolar fluid (see Fig. 1). The origin of the coordinate system is located at the centre of the cavity with x' and y' being the horizontal and vertical coordinates, respectively. The cavity is insulated on the sides and heated from the bottom by a uniform heat flux q'. All the boundaries of the cavity, except the upper free surface, are supposed to be rigid and impermeable. The upper free surface is assumed to be flat and subjected to a surface tension σ which varies linearly with temperature as $\sigma=\sigma_0[1-\gamma_T(T'-T'_0)]$ where the subscript 0 refers to conditions at a reference state and γ_T is the thermal surface tension gradient. The micropolar fluid is assumed to satisfy the Boussinesq approximation. The density variation with temperature is described by the state equation $\rho=\rho_0[1-\beta'_T(T'-T'_0)]$ where ρ_0 is the fluid mixture density at temperature $T'=T'_0$ and β'_T is the thermal expansion coefficient, respectively.

The governing equations, which describe the system behavior are conservation of momentum, microrotation and energy, are given below in terms of the stream function Ψ as (see for instance [19]):

$$\frac{\partial\nabla^2\Psi}{\partial t}+J(\Psi,\nabla^2\Psi)=\Pr(1+K)\nabla^2(\nabla^2\Psi)+\Pr K\nabla^2 N-\Pr Ra\frac{\partial T}{\partial x} \tag{1}$$

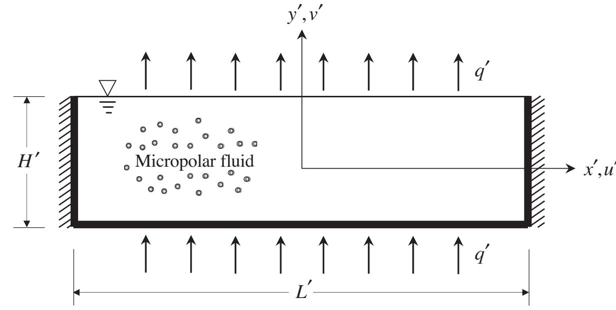

Fig. 1. Schematic diagram of the problem domain and coordinate system.