International Journal of Heat and Mass Transfer 54 (2011) 2774–2781

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Vascularization for cooling and mechanical strength

E. Cetkin [a], S. Lorente [b], A. Bejan [a,*]

[a] Duke University, Department of Mechanical Engineering and Materials Science, Durham, NC 27708-0300, USA
[b] Université de Toulouse, UPS, INSA, LMDC (Laboratoire Matériaux et Durabilité des Constructions), 135, Avenue de Rangueil, F-31 077 Toulouse Cedex 04, France

ARTICLE INFO

Article history:
Received 5 January 2011
Received in revised form 14 February 2011
Accepted 14 February 2011

Keywords:
Vascular
Constructal
Dendritic
Tree shaped
Radial channels
Cooling
Strength

ABSTRACT

In this paper we explore the use of vascular design that provides cooling and mechanical strength at the same time. We illustrate the concept with a circular plate vascularized with embedded channels. The cooling fluid enters to the plate from the center or from the rim, and leaves after it cools the plate down to an allowable temperature level. The vascular cooling channels also affect the mechanical strength of the plate. We simulated numerically the thermofluid and mechanical behavior for three different structures; radial, dendrites with one pairing level and dendrites with two pairing levels. We found that for a given set of conditions (applied pressure difference, coolant inlet position, and number of the cooling channels) there is one configuration that is best; however, there is no single configuration that is best for all conditions.

1. Introduction

Cooling and mechanical strength requirements place limits on the improvement of many advanced technologies, from electronic packages to structures for future air vehicles. Although the strength of the structure can be increased by adding more and stronger material, the challenge is to reduce the volume and weight of the structure. At the same time, the structure must be able to resist the effect of sudden heating, volumetrically or on its envelope. The design challenge is to increase structural strength while decreasing the maximum temperature in the volume, and reducing the volume.

Even though these objectives seem to be in conflict, the "flow of stresses" concept of constructal theory shows how to achieve mechanical strength by morphing the design to meet these two objectives and using less material [1]. The approach is to allow the structure to change freely by placing the material in places where stresses flow with fewer strangulations. This concept is the same as in the constructal design of thermofluid architectures: the design is free to change such that more flow volume is placed where it is needed. The current literature focuses mostly on thermofluid design [2–9] and smart features such as self-cooling and self-healing [10–15], and in a few instances mechanical strength was used as a design objective [16,17].

In the present study the flow volume houses three types of flows at the same time: stresses, fluid, and heat. We seek architectures that facilitate these three flows at the same time. The emerging designs are "designed porous structures" that consist of radial and tree-shaped cooling channels.

2. Hess–Murray's law: temperature dependence

Dendritic flow structures offer less resistance when bifurcations are accompanied by optimal step-changes in diameters, in accord with the Hess–Murray rule (e.g. Ref. [1]). This design rule applies to isothermal systems, and to fluids with temperature independent properties. Here we review concept by assuming the more general case of temperature dependent properties. Additional generality is provided by the assumption that the mother tube is continued by n identical daughter tubes, and by the fact that the flow regime can be laminar or turbulent. The fluid volume is fixed, and it is used as constraint in the search for the optimal mother/daughter diameter ratios. The pressure drop formula for laminar flow is

$$\Delta P = C\nu_i \dot{m}_i \frac{L_i}{D_i^4} \qquad (1)$$

where $C = 128/\pi$, and ν_i is the kinematic viscosity corresponding to the mean temperature $T_{me} = \int T\,dV/\int dV$. If one tube branches into n identical tubes, the total flow volume and pressure drop are

$$V = \pi\frac{D_1^2}{4}L_1 + n\pi\frac{D_2^2}{4}L_2 \qquad (2)$$

* Corresponding author.
E-mail address: abejan@duke.edu (A. Bejan).

0017-9310/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2011.02.061

Nomenclature

C	laminar flow pressure drop constant
C_T	turbulent flow pressure drop constant
C_{ijkl}	elasticity tensor, Eq. (15)
c_P	specific heat at constant pressure, J kg^{-1} K^{-1}
D_1, D_2	duct diameter, Eq. (2)
d_m	diameter of the cooling ducts, m, Figs. 1, 5 and 7
E	elasticity modulus
f	friction factor
F	volume forces, N
H	plate thickness, m, Fig. 1
k	thermal conductivity
K	second order tensor, Eq. (19)
L	duct length, m
$\dot{m}$	mass flow rate, kg s^{-1}
$\dot{\tilde{m}}$	volumetric flow rate, m^3 s^{-1}
n	number of the cooling ducts
P	pressure, N m^{-2}
Pr	Prandtl number, Eqs. (31)–(33)
q''	heat flux, W m^{-2}
R	plate radius, m, Figs. 1, 5 and 7
r, r_1, r_2	the distance from the center of the plate to the pairing junctions, m, Figs. 5 and 7
r_x, r_y, r_z	displacement, m
T	temperature, K
u, v, w	velocity components, m s^{-1}
V	volume
W	normal velocity of fluid enters from the inlet or leaves from the outlet, m s^{-1}
x, y, z	coordinates, m

Greek symbols

α	thermal diffusivity, m^2 s^{-1}
γ	Poisson ratio, Eq. (19)
ΔP	pressure difference
ε_{kl}	strain tensor, Eq. (15)
μ	dynamic viscosity, kg m^{-1} s^{-1}
ν	kinematic viscosity, m^2 s^{-1}
ρ	density, kg m^{-3}
σ	normal stress, N m^{-2}
σ_{ij}	stress tensor, Eqs. (14) and (15)
τ	shear stress, N m^{-2}
ϕ	shear strain, Eq. (21)

Subscript

i	rank of ducts
in	inlet
m	cooling duct pattern
max	maximum
me	mean
ref	reference
s	solid
st	mechanical
x, y, z	coordinates

Superscript

$\sim$	dimensionless

$$\Delta P = C\nu_1\dot{m}_1\frac{L_1}{D_1^4} + C\nu_2\dot{m}_2\frac{L_2}{D_2^4} \tag{3}$$

where $\dot{m}_2 = \dot{m}_1/n$. The diameter ratio for minimum ΔP is

$$\frac{D_1}{D_2} = n^{1/3}\left(\frac{\nu_1}{\nu_2}\right)^{1/6} \tag{4}$$

The case of fully developed turbulent flow is analyzed similarly. The pressure drop formula for turbulent flow is

$$\Delta P = \frac{C_T\dot{m}_i^2}{\rho_i}\frac{L_i}{D_i^5} \tag{5}$$

where $C_T = 32f/\pi^2$, f is the friction factor which is constant in the fully rough regime, and ρ_i is the density of the fluid corresponding to T_{me}. The total fluid volume and pressure drop are

$$V = \pi\frac{D_1^2}{4}L_1 + n\pi\frac{D_2^2}{4}L_2 \tag{6}$$

$$\Delta P = \frac{C_T\dot{m}_1^2}{\rho_1}\frac{L_1}{D_1^5} + \frac{C_T\dot{m}_2^2}{\rho_2}\frac{L_2}{D_2^5} \tag{7}$$

where $\dot{m}_2 = \dot{m}_1/n$. The optimal diameter ratio is

$$\frac{D_1}{D_2} = n^{3/7}\left(\frac{\rho_2}{\rho_1}\right)^{1/7} \tag{8}$$

In summary, the effect of variable properties is felt through the ratios $(\nu_1/\nu_2)^{1/6}$ and $(\rho_2/\rho_1)^{1/7}$ for laminar flow and turbulent flow, respectively. In the following work, the temperature variations are assumed to be small enough so that at every branching point the ratios $(\nu_1/\nu_2)^{1/6}$ and $(\rho_2/\rho_1)^{1/7}$ are sufficiently close to 1.

3. Model

We relied on a numerical model of the thermofluid and mechanical behavior of the circular plate. The diameter and thickness of the plate are D and H, and their ratio is fixed $D/H = 10$. The total volume is fixed, and so is the volume of the channels. The plate is subjected to uniformly distributed force and uniform heat flux, both acting from below, Fig. 1. Because the plate is thin, the heating from below is also an adequate approximation of situations where the heating is distributed volumetrically. The fluid flow is governed by the mass conservation and momentum equations, which for incompressible and steady flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{9}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \tag{10}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \tag{11}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \tag{12}$$

Here x, y and z are the coordinates, u, v and w are the velocity components corresponding to these coordinates, and P, ν and ρ are the pressure, kinematic viscosity and fluid density. The temperature distribution is found by solving the energy equation

$$\rho c_P\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{13}$$

where c_P is the specific heat at constant pressure, T is the temperature, and k is the fluid thermal conductivity.