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Effect of power-law fluid behavior on momentum and heat transfer characteristics of an inclined square cylinder in steady flow regime

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ABSTRACT

The governing equations describing the momentum and heat transfer phenomena of power-law non-Newtonian fluids over a heated square cylinder at 45° of incidence in the two-dimensional (2-D) steady flow regime are solved numerically. Extensive results on the detailed structure of the flow and temperature fields as well as on the gross engineering parameters are presented over the following ranges of conditions: $0.2 \leqslant n \leqslant 1$; $0.1 \leqslant Re \leqslant 40$ and $0.7 \leqslant Pr \leqslant 100$. At low Reynolds numbers, the flow remains attached to the surface of the cylinder. This seems to occur for all values of power-law index, at least up to about $Re = 1$. On the other hand, twin standing vortices were seen to form at $Re = 10$ for all values of power-law index considered herein. The influence of the Reynolds number and power-law index is delineated on the detailed structure of the flow field (streamlines), wake characteristics and surface pressure distribution as well as on the value of drag coefficients. Similarly, the effect of Prandtl number is studied on forced convective heat transfer for the two commonly encountered boundary conditions, namely, constant temperature or constant heat flux prescribed on the surface of the cylinder. Using the computed numerical results, simple heat transfer correlations are obtained in terms of the Nusselt number as a function of the pertinent governing parameters thereby enabling the prediction of the rate of heat transfer between the fluid and the immersed cylinder. In addition, variation of the local Nusselt number on the surface of the inclined of square cylinder and representative isotherm plots are also presented to elucidate the effect of Reynolds number, Prandtl number and power-law index on the heat transfer phenomenon.

1. Introduction

Owing to the wide occurrence of non-Newtonian fluid behavior in numerous industrial settings including polymer, food, pharmaceutical and personal care product manufacturing sectors, and allied processing applications [1–4], over the past 15–20 years, significant research effort has been expended in elucidating the role of non-Newtonian characteristics on momentum and heat transfer phenomena for bluff bodies of various shapes submerged in moving fluids. Generally, most macromolecular (polymer melts and their solutions) and multi-phase systems (foams, emulsions, pastes and suspensions, for instance) exhibit a spectrum of non-Newtonian features including shear-dependent viscosity, visco-elasticity, time-dependence, etc., under appropriate conditions. It is readily acknowledged that shear-thinning is probably the most common type of non-Newtonian fluid behavior encountered in industrial applications. The effective or apparent viscosity (shear stress divided by shear rate) of a shear-thinning substance can decrease by several orders of magnitude from a very high value at low shear rates (relevant to rest or storage conditions) to a vanishingly small value at high shear rates such as that encountered in pipe and pump flows, in mixing vessels and in bluff body flows. Obviously, the two-dimensional flow over a cylinder (irrespective of its cross-section) gives rise to a flow field in which the effective rate of deformation varies from point to point in a complex fashion. Conversely, unlike in the case of a Newtonian fluid whose viscosity is independent of the shear rate, the effective viscosity of a shear-thinning fluid can vary enormously in the vicinity of the bluff body depending upon the local value of the rate of deformation tensor. Needless to say, this in turn, is expected to have significant influence on the detailed structure of the velocity and temperature fields as well as on the gross parameters of engineering significance such as the Nusselt number, drag coefficient and wake phenomena, etc. Therefore, the interest in studying such model configurations is not only of intrinsic theoretical relevance, but it is also of overwhelming pragmatic significance such as in the use of tubes of various cross-sections in tubular, pin-type and in other novel designs of compact heat exchangers, in novel designs of mixing impellers, and in rake filters used for non-Newtonian slurries.

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Nomenclature

B	side of the square cylinder (m)
C_D	drag coefficient, $C_D = \frac{2F_D}{\rho U_\infty^2 D}$
C_{DF}	friction component of drag coefficient, $C_{DF} = \frac{2F_{DF}}{\rho U_\infty^2 D}$
C_{DP}	pressure component of drag coefficient, $C_{DP} = \frac{2F_{DP}}{\rho U_\infty^2 D}$
C	specific heat of the fluid (J kg^{-1} K^{-1})
C_p	pressure coefficient, $C_p = \frac{p-p_\infty}{\frac{1}{2}\rho U_\infty^2}$
D	projected length of cylinder in streamwise direction (m)
F_D	drag force per unit length of the cylinder (N m^{-1})
F_{DF}	frictional drag force per unit length of the cylinder (N m^{-1})
F_{DP}	pressure drag force per unit length of the cylinder (N m^{-1})
H	height of the domain (m)
I_2	second invariant of the rate of deformation tensor (s^{-2})
k	thermal conductivity of fluid (W m^{-1} K^{1})
L_r	wake length, measured from the center of the cylinder $L_r = (\frac{L_r'}{D})$
m	power-law consistency index (Pa s^n)
N_p	number of grid points on each face of square cylinder
Nu	local Nusselt number
Nu_{avg}	average Nusselt number
n	power-law flow behavior index
p	Pressure (Pa)
Pr	Prandtl number, $Pr = \frac{C_p m}{k}(\frac{U_\infty}{D})^{n-1}$
q_w	constant heat flux on the surface of the cylinder, (W m^{-2})
Re	Reynolds number, $Re = \frac{\rho U_\infty^{2-n} D^n}{m}$
T	dimensionless temperature, $T = \frac{(T'-T_\infty)}{(T_w-T_\infty)}$ or $T = \frac{(T'-T_\infty)}{q_w D/k}$
U_x, U_y	x- and y-components of velocity (m s^{-1})
U_∞	free stream velocity (m s^{-1})
x, y	streamwise and transverse coordinates (m)
X_d	downstream length (m)
X_u	upstream length (m)

Greek symbols

α	angle of incidence, degree
ε	component of the rate of the strain tensor (s^{-1})
φ	independent variable
η	viscosity (Pa s)
ρ	density of the fluid (kg m^{-3})
τ	extra stress tensor (Pa)
ψ	stream function (m^2 s^{-1})

Superscript

$'$	dimensional variable

Subscripts

∞	inlet condition
w	surface of the cylinder

Abbreviations

CHF	constant heat flux
CWT	constant wall temperature

While numerous models of varying forms and complexities are available in the literature which purport to approximate the shear-thinning behavior [1,2], the simple two parameter power-law model has gained wide acceptance in the literature [5]. This work is thus concerned with the flow of power-law fluids over a cylinder of a square cross-section with one of its corners aligned with the direction of flow (45° incidence) and with its long axis oriented transverse to the flow (as shown schematically in Fig. 1). In particular, this work is concerned with the elucidation of the influence of shear-thinning viscosity both at microscopic and macroscopic levels in the flow over a square cylinder with 45° incidence. However, it is instructive to briefly review the pertinent literature for the flow of Newtonian fluids past a square cylinder (especially with 45° incidence) and that of power-law fluids over cylinders of various cross-sections as this will subsequently facilitate the detailed presentation and discussion of the new results gleaned in this work.

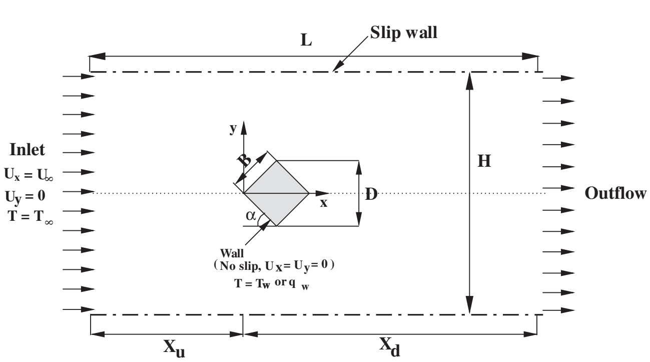

Fig. 1. Schematic representation of the flow past an inclined square cylinder.

2. Previous literature

The flow of fluids, especially Newtonian, over bluff bodies of various shapes including circular, square, elliptic and triangular cylinders, of axisymmetric shapes like spheres and spheroids, for instance, has been studied well over 100 years now. Consequently, a wealth of information has accrued on various aspects of momentum, heat and mass transfer phenomena over wide ranges of conditions, especially for a circular cylinder and a sphere [6–9]. In contrast, though a reasonable body of knowledge for other shapes like elliptic, triangular and square cylinders is now available, but it is neither as extensive nor as coherent as that for the circular cylinder. In spite of the significant influence of the bluff body shape on flow dynamics, one can discern some common underlying characteristics. For instance, for the simplest case of the unconfined flow past a circular cylinder, depending upon the value of the Reynolds number, the flow undergoes several transitions from one flow regime to another. Thus, for instance, at very low Reynolds numbers, since the fluid inertia is negligible, fluid elements are able to negotiate the shape of the submerged bluff body and thus closely follow its contours, i.e., the flow remains attached to the surface. As the Reynolds number is gradually incremented, fluid inertia increases and a point is reached where it leads to the occurrence of the so-called adverse pressure gradient at some point on the surface of the bluff body. This causes the flow to detach itself and thereby leading to the appearance of the separation bubble at about $Re \approx 4-5$. With further increase in the value of the Reynolds number, the separation bubble grows in size until the wake becomes asymmetric (about the mid-plane) and finally, it becomes unstable at about $Re \approx 47-48$. Beyond this value of the Reynolds number, the vortices are shed and the flow field becomes periodic in time, albeit it is still two-dimensional and the wake is laminar. At about $Re \approx 161-200$, the wake itself becomes turbulent and the overall flow field exhibits some features of three-dimensional