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Gas slippage effect on the permeability of circular cylinders in a square array

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ABSTRACT

The gas slippage phenomenon is usually observed when gas flow through porous media under low pressure, and has received an increasing attention for its importance in science and engineering. In the past decades, although many theoretical, numerical and experimental works have been done to predict the permeability of a square array of circular cylinders, little literature is available in studying the slippage effect on permeability. In this paper, the gas slippage effect on the permeability of circular cylinders in a square array is studied theoretically. The permeabilities at low and high solid fractions are derived with cell model and lubrication theory by coupling with a first-order slip boundary condition. The present results are finally validated through a comparison with some available results.

1. Introduction

Gas flow in porous media has been one of the most important topics due to its wide applications in both science and engineering [1–3]. Usually, slow gas flows through porous media are assumed to obey the celebrated Darcy law,

$$U = -\frac{K}{\mu}\nabla P, \tag{1}$$

where U is the superficial velocity, μ is the dynamic viscosity, ∇P is the pressure gradient in the direction of flow, K is the permeability. In practice, permeability, as one of the most important physical parameters for flow through porous media, must be known before we use Darcy law. Therefore, how to derive the permeability becomes a key problem in studying gas flow through porous media. Due to the complex structures of porous media, however, most results on permeability are empirical correlations [1,3]. To derive analytical expressions on permeability, some ideal porous media have been proposed in the past years [1–4]. Circular cylinders in a square array (see Fig. 1), as a special case in these ideal models, obtains an increasing interest for its simplicity and usefulness in practice [2–4].

In the year 1959, Happel analytically solved the Stokes equation for flow normal to circular cylinders by using a simple approximation called a free surface model (or cell model appeared in later publications) and zero shear stress boundary condition at the perimeter of circular cylinder [5]. The permeability in Happel's work can be written as

$$\frac{K}{D_p^2} = \frac{1}{32\phi}\left(-\ln\phi + \frac{\phi^2-1}{\phi^2+1}\right), \tag{2}$$

where D_p is the diameter of circular cylinder, ϕ is cylinder density or solid fraction. Almost at the same time, Kuwabara also solved this problem with a similar approach, but the zero vorticity instead of zero shear stress boundary condition is used [6]. In Kuwabara's paper, the author presented an analytical solution of drag force per unit length which is equivalent to permeability as

$$\frac{K}{D_p^2} = \frac{1}{32\phi}\left(-\ln\phi - \frac{3}{2} + 2\phi - \frac{1}{2}\phi^2\right). \tag{3}$$

Hasimoto used Fourier series to obtain another analytical solution of this problem in the same year [7]. The drag force in Hasimoto's work is related to permeability

$$\frac{K}{D_p^2} = \frac{1}{32\phi}(-\ln\phi - 1.476 + 2\phi). \tag{4}$$

In the year 1982, Sangani and Acrivos extended Hasimoto's approach to derive a new theoretical solution of drag force [8]. The permeability in their work can be expressed as

$$\frac{K}{D_p^2} = \frac{1}{32\phi}(-\ln\phi - 1.476 + 2\phi - 1.774\phi^2 + 4.076\phi^3). \tag{5}$$

Later, Drummond and Tahir used a method of singularities to present a more accurate solution on drag force, from which the permeability is obtained [9]

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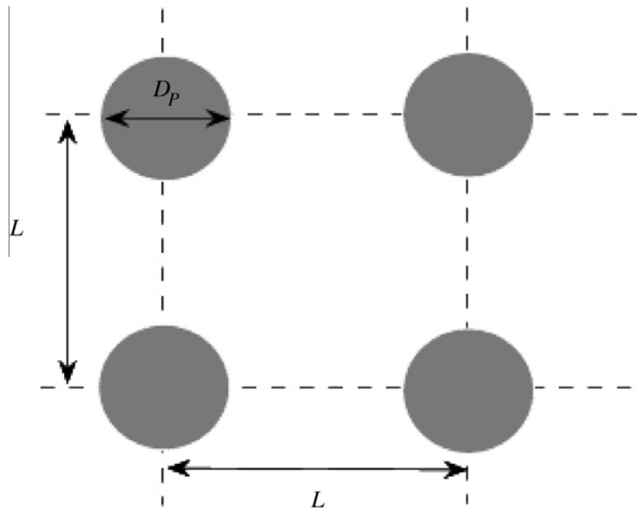

Fig. 1. The schematic of circular cylinders in a square array.

$$\frac{K}{D_p^2} = \frac{1}{32\phi}\left(-\ln\phi - 1.47633597 + \frac{2\phi - 0.79589781\phi^2}{1 + 0.48919241\phi - 1.60486942\phi^2}\right). \quad (6)$$

Up to now, there are a considerable theoretical results on permeability of circular cylinders in a square array [5–8], and these results also have been validated numerically and experimentally [4,8]. It should be noted that, however, above theoretical solutions [Eqs. (2)–(6)] on permeability are limited to low solid fraction cases (see Fig. 2). In order to present another limit result to the case with high solid fraction, the lubrication theory, as an asymptotic approach, has been used by some authors. For example, Keller [10] first presented an asymptotic solution on drag force with the lubrication theory, the dimensionless permeability can be written as

$$\frac{K}{D_p^2} = \frac{\sqrt{2}}{18\phi}\left(1 - 2\frac{\sqrt{\phi}}{\pi}\right)^{\frac{5}{2}}, \quad (7)$$

which is also reported in a later work [8]. Bruschke and Advani derived a more strict result with the same method, the dimensionless permeability in their work can be expressed explicitly as [11]

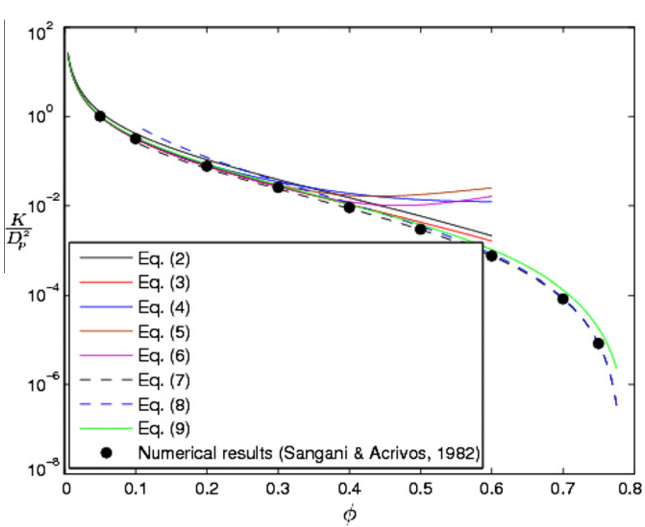

Fig. 2. Theoretical results on the permeability of circular cylinders in a square array.

$$\frac{K}{D_p^2} = \frac{(1-\iota^2)^2}{12\iota^3}\left(1 - \frac{1}{2}\iota^2 + \frac{3\iota}{\sqrt{1-\iota^2}}\arctan\sqrt{\frac{1+\iota}{1-\iota}}\right), \quad (8)$$

where $\iota = 2\sqrt{\phi/\pi}$. As seen from Fig. 2, it is clear that Eqs. (7) and (8) indeed present a good prediction on permeability at high solid fraction. However, none of them (the results derived from cell model or lubrication theory) can successfully predict permeability at moderate solid fraction. To overcome the limits of above results, Bruschke and Advani also presented a hybrid model on permeability [11],

$$\frac{K}{D_p^2} = A_{CM}K_{CM} + A_{LT}K_{LT}, \quad (9)$$

where $A_{CM} = 1 - e^{\tau(1-\phi_{\max}/\phi)}, A_{CM} = 1 - e^{\tau[1-\phi_{\max}/(\phi_{\max}-\phi)]}$ are combination coefficients with $\phi_{\max} = \pi/4$, $\tau = 0.8$. K_{CM} and K_{LT}, defined by Eqs. (3) and (8), are limit theoretical results derived from the cell model and lubrication theory. The result based on the hybrid model is also shown in Fig. 2. As seen from this figure, the hybrid model presents a good prediction on permeability over all solid fractions, but a relatively little poor prediction for the case with low solid fraction is also observed.

All of above stated studies focus on continuum gas flow through a square array of circular cylinders ($Kn < 10^{-3}$, $Kn = 2\lambda/D_p$ is the Knudsen number), as a result, the gas slippage effect is neglected. However, with the development of air filtration, petroleum and natural gas industries [1,2,12], the gas slippage phenomena induced by gas slip on solid surface, as an important abnormal transport phenomena, has attained an increasing attention [2,12–15]. Pich may be the first one who investigated gas through fibrous filters with a first-order slip boundary condition, the author obtained a theoretical result on pressure drop by using cell model developed by Kuwabara, the equivalent permeability can be derived [13]

$$\frac{K}{D_p^2} = \frac{1}{32\phi(1+1.996Kn)}\left[-\ln\phi - \frac{3}{2} + 2\phi - \frac{1}{2}\phi^2 + 1.996Kn\left(-\ln\phi - \frac{1}{2} + \frac{1}{2}\phi^2\right)\right], \quad (10)$$

some similar results are also obtained in a later work [14]. The same as described previously, Eq. (10) is limited to cases with low solid fraction since it is derived from the cell model. In addition, we also found that, as $Kn \to 0$, Eq. (10) will reduce to Eq. (3) which is just the result without slippage effect. Recently, Wang studied this problem with a semi-numerical method, and derived a series solution of permeability [15,16], but the results derived by Wang is also limited to special inclusions. Chai et al also numerically studied the same problem with lattice Boltzmann method, but a second-order slip boundary condition is adopted in their work [12].

Through a survey of available literature on the permeability with gas slippage effect, no theoretical work on the permeability at high solid fraction is found. In this work, we will present a systematically theoretical study on permeability with gas slippage effect, which can be viewed as an extension to available results. The paper is organized as follows. In the following Section 2, the permeabilities at low and high solid fractions are derived respectively with cell model and lubrication theory by using slip boundary condition, and main attention is given to the case with high solid fraction. In Section 3, we present a hybrid model that can predict permeabilities over all solid fractions and some discussions on the permeability with slippage effect. Finally, some conclusions are given in Section 4.

2. Theoretical results on permeability with gas slippage effect

In this section, we will present a theoretical study on permeability with gas slippage effect. Following the approaches used in