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Thermal-diffusion and diffusion-thermo effects on axisymmetric flow of a second grade fluid

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ABSTRACT

This study investigates the thermal-diffusion and diffusion-thermo effects on the two-dimensional magnetohydrodynamics (MHD) axisymmertric flow of a second grade fluid. Mathematical analysis has been carried out in the presence of Joule heating and first order chemical reaction. Using momentum, energy and concentration laws, the governing partial differential equations have been reduced to the ordinary differential equations by suitable transformations. Series solutions are constructed by homotopy analysis method (HAM). Convergence of the derived series solutions is ensured. Plots are displayed in order to examine the influence of emerging parameters on the dimensionless components of velocity, temperature and concentration fields. Numerical computations for skin friction coefficient, Nusselt number and Sherwood number are tabulated.

1. Introduction

Recently, the stretching flows have received considerable attention of the researchers. This is due to their demands in aerodynamic extrusion of plastic sheets, cooling of an infinite plate in a cooling bath, liquid film in condensation process, continuous filament extrusion from a dye, the fluid dynamic of a long thread traveling between a feed roll and wind-up roll etc. Many investigations regarding stretching flows are already in different configurations. This topic has been given proper attention. However a fewer studies on axisymmetric flow driven by stretching surface are also available. Some recent contributions relevant to axisymmetric flows induced by stretching surface have been discussed in the studies [1–8].

It is now well established fact that energy and mass fluxes are engendered by composition (concentration) and temperature gradients, respectively. In fact Dufour effect (diffusion-thermo) signifying energy transfer occurs because of concentration gradient. However in view of temperature gradient there is a Soret effect (thermal-diffusion) which leads to mass transfer. The Dufour and Soret effects can not be neglected when one considers the flows of mixture of gases with light molecular weights (He, H_2) and of moderate molecular weights (N_2, air). Such facts are mentioned in the studies [9–16] which investigate the Dufour and Soret effects on heat and mass transfer respectively. Some other notable investigations [18–22] highlight the Dufour and Soret effects regarding the flows induced by moving boundaries. Moreover, it is noticed that the chemical reaction may affect (retard/enhance) the mass transfer of diffusing species. The absorption of carbon dioxide into carbonate–bicarbonate buffer solution in the presence of an arsenite ion catalysts is an example of first order chemical reaction [23]. Further, the recent studies [24,25] have examined the effect of mass transfer on the flow of non-Newtonian fluids in the presence of first order chemical reaction.

Literature survey reveals that no attempt regarding Dufour and Soret effects on second grade fluid flow induced by radially stretching sheets has been made so far. Note that mixture of polymethyl methacrylate in *n*-butyl acetate and pyridine at 25 °C containing 30.5 g of polymer per litre behaves very nearly as the second grade fluid. Isobutylene is an other example of a second grade fluid. In recent communication [2] it has been proved that solution exist for positive values of second grade parameter α. Therefore positive values of α are taken (see figures and tables). The aim of present work is to investigate the effects of diffusion-thermo and thermal-diffusion on steady laminar MHD flow driven by two radially stretching sheets. The first order chemical reaction is also included. The governing problems are solved analytically by homotopy analysis method (HAM). This method is a powerful technique and has been used by many researchers in the recent studies [26–37].

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2. Mathematical analysis

We consider axisymmetric flow of an electrically conducting second grade fluid between two infinite parallel radially stretching sheets placed at $z = \pm L$. The flow is induced by the linear radial stretching of two sheets. The flow is considered symmetric about $z = 0$. A uniform magnetic field $\mathbf{B}_o$ perpendicular to the planes of sheets is applied i.e. in the z-direction. It is assumed that the magnetic Reynolds number is very small and induced magnetic field is neglected. There is no external electric field. Both the sheets have constant temperature T_w and constant concentration field C_w. Furthermore, temperature and concentration fields in the centre region between sheets are smaller than T_w and C_w, respectively. Physical model and coordinate system are given in Fig. 1. The velocity field for the flow under consideration is given by

$$\mathbf{V} = [u(r,z),\, 0,\, w(r,z)], \tag{1}$$

where u and w are the velocity components in the radial and axial directions.

By virtue of above definition of velocity, the governing equations are:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{2}$$

$$\begin{aligned} u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = {} & -\frac{1}{\rho}\frac{\partial p}{\partial r} + v\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} - \frac{u}{r^2}\right] \\ & + \frac{\alpha_1}{\rho}\left[\begin{array}{c} \frac{2u^2}{r^3} - \frac{2w}{r^2}\frac{\partial u}{\partial z} - \frac{1}{r}\left(\frac{\partial u}{\partial z}\right)^2 \\ -\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2} + w\frac{\partial^3 u}{\partial z^3} - \frac{2u}{r^2}\frac{\partial u}{\partial r} + \frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial z^2} \\ + \frac{\partial w}{\partial r}\frac{\partial^2 w}{\partial z^2} + \frac{1}{r}\left(\frac{\partial w}{\partial z}\right)^2 + \frac{2w}{r}\frac{\partial^2 u}{\partial r \partial z} \\ +2\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial r \partial z} - \frac{\partial u}{\partial r}\frac{\partial^2 w}{\partial r \partial z} + u\frac{\partial^2 u}{\partial r \partial z^2} \\ +w\frac{\partial^3 w}{\partial r \partial z^2} + \frac{2u}{r}\frac{\partial^2 u}{\partial r^2} + 2\frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial r^2} \\ + \frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial r^2} + 2\frac{\partial w}{\partial r}\frac{\partial^2 w}{\partial r^2} + 2w\frac{\partial^3 u}{\partial r^2 \partial z} \\ +u\frac{\partial^3 w}{\partial r^2 \partial z} + 2u\frac{\partial^3 u}{\partial r^3} \end{array}\right] - \frac{\sigma B_o^2}{\rho}u, \end{aligned} \tag{3}$$

$$\begin{aligned} u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = {} & -\frac{1}{\rho}\frac{\partial p}{\partial z} + v\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right] \\ & + \frac{\alpha_1}{\rho}\left[\begin{array}{c} \frac{w}{r}\frac{\partial^2 u}{\partial z^2} - \frac{1}{r}\frac{\partial u}{\partial z}\frac{\partial w}{\partial z} + 2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2} \\ +2\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial z^2} + 2w\frac{\partial^2 w}{\partial z^3} + \frac{1}{r}\frac{\partial u}{\partial r}\frac{\partial u}{\partial z} \\ + \frac{1}{r}\frac{\partial w}{\partial r}\frac{\partial w}{\partial z} + \frac{\partial w}{\partial r}\frac{\partial^2 u}{\partial z^2} - \frac{1}{r}\frac{\partial u}{\partial r}\frac{\partial w}{\partial r} \\ + \frac{u}{r}\frac{\partial^2 u}{\partial r \partial z} - \frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial r \partial z} + 2\frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial r \partial z} \\ + \frac{w}{r}\frac{\partial^2 w}{\partial r \partial z} + \frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial r \partial z} + w\frac{\partial^3 u}{\partial r \partial z^2} \\ +2u\frac{\partial^3 w}{\partial r \partial z^2} + \frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial r^2} - \frac{\partial w}{\partial r}\frac{\partial^2 u}{\partial r^2} + \frac{u}{r}\frac{\partial^2 w}{\partial r^2} \\ + \frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial r^2} + u\frac{\partial^3 u}{\partial r^2 \partial z} + w\frac{\partial^3 w}{\partial r^2 \partial z} + u\frac{\partial^3 w}{\partial r^3} \end{array}\right], \end{aligned} \tag{4}$$

$$\begin{aligned} & u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} \\ & = \frac{K}{\rho c_p}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{\mu}{\rho c_p}\left[\begin{array}{c} 2\frac{u^2}{r^2} + 2\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 \\ +2\frac{\partial w}{\partial r}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) + 2\left(\frac{\partial w}{\partial z}\right)^2 \end{array}\right] \\ & + \frac{\alpha_1}{\rho c_p}\left[\begin{array}{c} \frac{2u^2}{r^2}\frac{\partial u}{\partial r} - \frac{2u^3}{r^3} + 2u\frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial r^2} + 2w\frac{\partial u}{\partial r}\frac{\partial^2 u}{\partial r \partial z} + 2\frac{wu}{r^2}\frac{\partial u}{\partial z} + u\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial r \partial z} \\ +w\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial z^2} + u\frac{\partial w}{\partial r}\frac{\partial^2 u}{\partial r \partial z} + w\frac{\partial^2 u}{\partial z^2}\frac{\partial w}{\partial r} + u\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial r^2} + u\frac{\partial w}{\partial r}\frac{\partial^2 w}{\partial r^2} + w\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial r \partial z} \\ +w\frac{\partial w}{\partial r}\frac{\partial^2 w}{\partial r \partial z} + 2u\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial r \partial z} + 2w\frac{\partial w}{\partial z}\frac{\partial^2 w}{\partial z^2} \end{array}\right] \\ & + \frac{\sigma B_o^2}{\rho c_p}u^2 + \frac{DK_T}{c_p C_s}\left[\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2}\right], \end{aligned} \tag{5}$$

$$u\frac{\partial C}{\partial r} + w\frac{\partial C}{\partial z} = D\left[\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2}\right] + \frac{DK_T}{T_m}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right] - K_1 C, \tag{6}$$

where $\alpha_1(\geqslant 0)$ designates the material constant, T the temperature field, C the concentration field, $\rho$the density, v the kinematic viscosity, c_p the specific heat, σ the electrical conductivity of the fluid, p the pressure, K the thermal conductivity, D the coefficient of mass diffusivity, C_s the concentration susceptibility, T_m the mean fluid temperature, K_T the thermal-diffusion ratio and K_1 the chemical reaction constant.

The boundary conditions are:

$$\begin{aligned} & \frac{\partial u}{\partial z} = 0, \quad w = 0, \quad \frac{\partial T}{\partial z} = 0, \quad \frac{\partial C}{\partial z} = 0, \quad \text{at } z = 0, \\ & u = ar, \quad w = 0, \quad T = T_w, \quad C = C_w, \quad \text{at } z = L, \quad a > 0. \end{aligned} \tag{7}$$

Employing:

$$u = arf'(\eta), \quad w = -2aLf(\eta), \quad \theta = \frac{T}{T_w}, \quad \phi = \frac{C}{C_w}, \quad \eta = \frac{z}{L}. \tag{8}$$

in Eqs. (2)–(7), one obtains:

$$\begin{aligned} & f''''(\eta) - Re\,Mf''(\eta) + 2Re\,f(\eta)f'''(\eta) - 2\alpha ff''''' = 0, \\ & f(0) = 0, \quad f(1) = 0, \quad f'(1) = 1, \quad f''(0) = 0, \end{aligned} \tag{9}$$

$$\begin{aligned} & \theta''(\eta) + 2Re\,Pr\,f(\eta)\theta'(\eta) + Pr\ Ec\left[(f'')^2 + 24\frac{1}{\delta}(f')^2\right] \\ & \quad + \alpha\ Pr\ Ec\left[f'(f'')^2 - 2ff''f''' - 24\frac{1}{\delta}ff'f''\right] \\ & \quad + Re\,M\ Pr\ Ec(f'(\eta))^2 + Du\ Pr\ \phi''(\eta) = 0, \\ & \theta'(0) = 0, \quad \theta(1) = 1, \end{aligned} \tag{10}$$

$$\begin{aligned} & \phi''(\eta) + 2Sc\,Re\,f(\eta)\phi'(\eta) + Sc\,Sr\,\theta''(\eta) - Re\,Sc\,\gamma\phi(\eta) = 0, \\ & \phi'(0) = 0, \quad \phi(1) = 1, \end{aligned} \tag{11}$$

where

$$\begin{aligned} & \alpha = \frac{\alpha_1 a}{\mu}, \quad Re = \frac{aL^2}{v}, \quad M = \frac{\sigma B_0^2}{\rho a}, \quad Pr = \frac{\mu c_p}{K}, \quad Du = \frac{DK_T C_w}{v C_s c_p T_w}, \\ & Sc = \frac{v}{D}, \quad Sr = \frac{DK_T T_w}{v T_m C_w}, \quad Ec = \frac{a^2 r^2}{c_p T_w}, \quad \gamma = \frac{K_1}{a}, \quad \delta = \frac{r^2}{L^2}, \end{aligned}$$

respectively denotes the second grade parameter, the Reynolds number (Re), Hartman number (M), Prandtl number (Pr), Dufour number (Du), Schmidt number, Soret number (Sr), local Eckert number (Ec), first order chemical reaction parameter (γ) and dimensionless length (δ). The dimensionless parameters Du and Sr correspond to Dufour and Soret effects respectively. It is evident from the expressions of Dufour and Soret numbers that these are arbitrary constants provided that their product remains constant. This fact is stated in the attempts [9–15]. Furthermore $Du = 0$ and $Sr = 0$ correspond to the situation when thermal diffusion and diffusion-thermo effects are smaller order of magnitude than the effects described by Fourier's and Fick's laws [9]. There is controversy on the sign of Soret number Sr. In some studies [9,10,14] $Sr(=DK_T(T_w - T_\infty)/vT_m(C_w - C_\infty))$ is considered as positive dimensionless parameter based on the fact that surface temperature T_w and surface concentration C_w are higher than the corresponding temperature and concentration at ambient fluid, i.e. $T_w > T_\infty$ and $C_w > C_\infty$. However some investigators [16,17] have taken S_r negative as well. In these attempts they have considered both the cases when $T_w > T_\infty$ and $C_w > C_\infty$ and $T_w > T_\infty$ and $C_w < C_\infty$. However in present study Sr is taken positive because all quantities in the expression