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Entransy expression of the second law of thermodynamics and its application to optimization in heat transfer process

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ABSTRACT

Based on theories of thermodynamics, the energy equation in terms of entransy in heat transfer process is introduced, which not only describes the change of entransy, but also defines the entransy consumption rate. According to the regularity of entransy change in heat transfer process and the effect of entransy consumption rate on the irreversibility of heat transfer process, it can be found that entransy is a state variable, from which a new expression for the second law of thermodynamics is presented. Then by setting entransy consumption rate and power consumption rate as optimization objective and constraint condition for each other, the Lagrange conditional extremum principle is used to deduce momentum equation, constraint equation and boundary condition for optimizing flow field of convective heat transfer, which are applied to simulate convective heat transfer coupling with energy equation in an enclosed cavity. Through the numerical simulation, the optimized flow field under different constraint conditions is obtained, which shows that the principle of minimum entransy consumption is more suitable than the principle of minimum entropy generation for optimizing convective heat transfer process.

1. Introduction

Past centuries have witnessed the gradual perfection of classical thermodynamics theories including irreversible thermodynamics. The second law of thermodynamics in particular, which is regarded as a fundamental law of physics, has found wide applications in engineering and scientific fields. However, in the field of heat transfer which is most closely related to thermodynamics, no important theoretical breakthrough has been achieved to heat transfer optimization over the past decades. The reason for this may be that the second law of thermodynamics has not been closely integrated with heat transfer theories in developing new concept and methodology.

In order to evaluate the degree of heat transfer enhancement, Guo afresh surveyed the mechanism for convective heat transfer and proposed the field synergy principle [1,2]. This novel concept attracted much attention of researchers [3–8]. Through their consistent effort, a systematic assessment for performance of heat transfer enhancement was gradually established. As we know that to make heat exchange equipment work efficiently with high heat transfer coefficient and lower flow resistance or lower energy consumption, the key factor is to optimize heat transfer process. For this purpose, Guo et al. newly proposed a new physical quantity "entransy" based on the analogy between heat conduction and electrical conduction as well as thermodynamics theories [9]. They deducted this concept theoretically and validated it through modeling and numerical computation [10–16]. Although introduction of the concept of entransy has shown the advantages in heat transfer optimization, it is still in its initial stage and needs to be perfected. By resorting to the second law of thermodynamics, this paper attempts to give new insights into the concept of entransy which is treated as a basic physical quantity, and to reaffirm the importance of entransy in developing optimization theories for heat transfer process.

2. The Irreversibility of heat transfer process

The transfer of energy, momentum and mass are three main forms of transport phenomena in the nature or engineering. Energy takes many forms, such as thermal energy, mechanical energy, electronic energy, optical energy, sound energy, etc. When energy in any form other than thermal energy is transferred, a part of it will be transformed into thermal energy, and when thermal energy is transferred, a certain amount of it will lose or dissipate to somewhere. This is the so-called irreversibility of energy conversion and transport processes. Therefore, in order to reduce the irreversibility of heat transfer process, it is necessary to explore its physical mechanism and optimize the process by quantitatively analyzing the changes of physical quantities and recording the traces left by these changes.

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Nomenclature

A, B, C_0	Lagrange multipliers
c	specific heat [J/(kg · K)]
c_p	specific heat at constant pressure [J/(kg · K)]
h	enthalpy [J/kg]
J, J'	functional
p	pressure [Pa]
Q	heat flux [W]
$\dot{Q}'''$	internal heat source [W/m^3]
$\boldsymbol{q}$	heat flux vector [W/m^2]
S	entropy [J/K]
s	specific entropy [J/(kg · K)]
$S_{g,p}$	entropy generation induced by fluid viscosity [J/K]
$S_{g,T}$	entropy generation induced by heat transfer [J/K]
T	temperature [K]
t	time [s]
$\boldsymbol{U}$	velocity vector [m/s]
V	volume [m^3]
Z	entransy [J · K]
z	specific entransy [J · K/kg]
$Z_{e,T}$	entransy consumption [J · K]
Greek symbols	
λ	thermal conductivity [W/(m · K)]
ρ	fluid density [kg/m^3]
μ	viscosity coefficient [kg/(m · s)]
Φ	dissipated heat from fluid viscosity [W/m^3]
Ω	control volume [m^3]

2.1. Thermodynamic definition of entransy and its deduction

For a non-equilibrium convective heat transfer process, its energy equation can be expressed in terms of enthalpy as:

$$\rho\frac{Dh}{Dt} = -\nabla\cdot\boldsymbol{q} + \Phi + \dot{Q}'''. \tag{1}$$

Where ρ is the fluid density, h is the fluid enthalpy, $-\nabla\cdot\boldsymbol{q}$ is the heat flux transferring into and out of fluid elementary volume, Φ is the dissipated heat from fluid viscosity, and $\dot{Q}'''$ is the internal heat source.

Eq. (1) can be rewritten in terms of entropy as:

$$\rho\frac{Ds}{Dt} = -\nabla\cdot\left(\frac{\boldsymbol{q}}{T}\right) + \frac{\lambda(\nabla T)^2}{T^2} + \frac{\Phi}{T} + \frac{\dot{Q}'''}{T}. \tag{2}$$

Where s is the entropy of fluid, $-\nabla\cdot(\frac{\boldsymbol{q}}{T})$ is the entropy flow transferring into and out of fluid elementary volume, $\frac{\lambda(\nabla T)^2}{T^2}$ is the entropy generation rate induced by heat transfer process, $\frac{\Phi}{T}$ is the entropy generation rate caused by the heat generation from viscous dissipation of mechanical energy, which can also be defined as analogical entropy source, and $\frac{\dot{Q}'''}{T}$ is the internal heat source in form of entropy expression. The differential entropy for incompressible fluid can be expressed as: $ds = \frac{dh}{T} = \frac{cdT}{T}$.

The principle of entropy generation minimization is widely utilized to optimize the thermodynamics process based on the second law of thermodynamics [17]. According to finite time thermodynamics theory [18], the entropy generation rate induced by heat transfer is a representation of the irreversibility of the process, so it can be denoted as the dot product of entropy flow and force, i.e. $\frac{\lambda(\nabla T)^2}{T^2} = \frac{\boldsymbol{q}}{T}\cdot(-\frac{\nabla T}{T})$, while the entropy generation rate $\frac{\Phi}{T}$ denotes mechanical energy dissipation induced by fluid viscosity. Accordingly, thermodynamic equilibrium of entropy is given as:

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{g,T} + S_{g,p}. \tag{3}$$

Where the integral term represents entropy flow, $S_{g,T}$ represents entropy generation induced by heat transfer, and $S_{g,p}$ represents entropy generation induced by fluid viscosity. When boundary heat flux is zero, entropy flow is zero. Then Eq. (3) reduces to:

$$\Delta S = S_{g,T} + S_{g,p}. \tag{4}$$

Multiplying both sides of Eq. (2) by T^2 and making transformation yields:

$$\rho cT\frac{DT}{Dt} = -\nabla\cdot(\boldsymbol{q}T) - \lambda(\nabla T)^2 + \Phi T + \dot{Q}'''T. \tag{5}$$

Referencing to the definition of entropy, the entransy of incompressible fluid, termed z, can be defined in the following differential expression:

$$dz = Tdh = cTdT. \tag{6}$$

It can be seen that Eq. (6) is a differential definition for entransy similar to thermodynamics entropy, which shows that the relation between differential enthalpy and temperature is expressed as product other than quotient. Thus the energy equation of convective heat transfer can be expressed in terms of entransy as:

$$\rho\frac{Dz}{Dt} = -\nabla\cdot(\boldsymbol{q}T) - \lambda(\nabla T)^2 + \Phi T + \dot{Q}'''T. \tag{7}$$

Where z is the entransy of fluid, $-\nabla\cdot(\boldsymbol{q}T)$ is the entransy flow transferring into and out of fluid elementary volume, $\lambda(\nabla T)^2$ is the entransy consumption rate induced by heat transfer process, ΦT is defined as the analogical entransy source induced by dissipated heat from fluid mechanical energy, and $\dot{Q}'''T$ is the internal heat source in form of entransy expression.

As we know that a state variable or parameter can be defined to represent the state of a thermodynamics system. If the values of all state variables of a system are known, the state of this system can be well described. The entransy is such a variable introduced to express the nature of a system, in which the heat is transferred from the high to the low temperature sites. The entransy consumption rate is induced by heat transfer temperature difference, which can

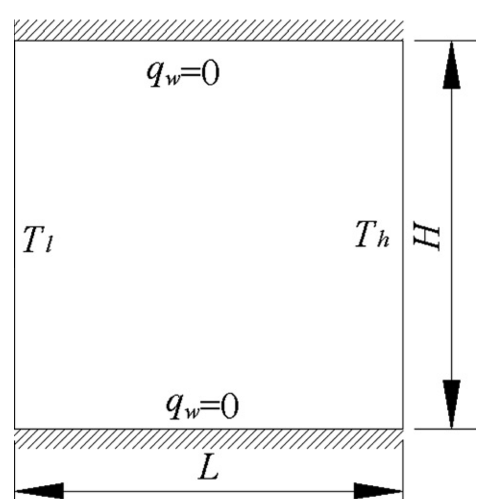

Fig. 1. Sketch of an enclosed cavity with geometry and boundary conditions (L = H = 15 mm).