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The onset of convection in porous layers with multiple horizontal partitions

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ABSTRACT

In this paper we investigate the onset of convection in a horizontally partitioned porous layer which is heated from below. Identical sublayers are separated by thin impermeable barriers. A linear stability analysis is performed, and dispersion relations are obtained directly and explicitly for two- and three-layer configurations. A systematic numerical procedure is devised to compute the dispersion relation for an arbitrary number of sublayers, but from this it is possible to guess the correct analytical form of the dispersion relation for general cases.

Neutral stability curves are found to organise themselves into natural groups of N members when there are N sublayers. When the disturbance wavenumber, k, is large, each member of any group lies within an $O(k^{-1})$ distance of all other members, but within an $O(1)$ distance of other groups. When the number of sublayers is large, the system tends towards one with a critical Darcy–Rayleigh number of 12 and a critical wavenumber of zero; this is the well-known property of a single porous layer with constant heat flux boundary conditions. An asymptotic analysis is performed in order explore these two apparently disparate configurations. Finally, another asymptotic analysis is used to determine the critical Rayleigh number and its associated wavenumber when the number of sublayers is large.

1. Introduction

In the present paper we consider the onset of convection within a horizontal porous layer heated from below which has multiple infinitesimally thin impermeable horizontal partitions embedded within it. The resulting sublayers are then identical in all respects, including their height. Thus the sublayers are mechanically decoupled but remain thermally coupled since the interfaces present no barrier to the conduction of heat. This work forms an extension to our previous paper [1] which was concerned with a two-layer system.

Many authors have considered how the presence of layering affects the onset of convection and the subsequent nonlinear cellular flow. Being motivated mostly by geothermal applications, these layered systems generally tend to have interfaces though which fluid may flow, unlike the topic of the present paper. The first person to consider layering was Georghitza [2] who considered weak layering in the sense that the difference in the permeabilities of the two layers was small. On the other hand, Donaldson [3] considered a two-layer system where one of the sublayers is impermeable and computed nonlinear two-dimensional flow patterns. Riahi [4] considered what might be termed a three-layer configuration where a porous layer is sandwiched between two impermeable but conducting regions of infinite height. He conducted a weakly nonlinear analysis and found that there is a region in parameter space within which two-dimensional rolls do not form the favoured convection pattern; this role is passed to a square-cell pattern. Further analyses of this type may be found in Mojtabi and Rees [5] and Rees and Mojtabi [6]. Masuoka et al. [7] provided some criteria for the onset of convection in a two-layer configuration where both sublayers are porous, and Rana et al. [8] conducted a numerical study of a three-layer configuration which was believed to model well the Pahoa reservoir in Hawaii.

A more systematic approach to these problems was provided in the 1980s by McKibbin and co-workers who provided comprehensive data on the onset problem [9], post-critical heat transfer [10], the effects of thin highly permeable cracks [11] and almost impermeable sheets [12]. Indeed, the present paper may also be regarded as an extension of [12] to the case where the sheets are completely impermeable. Jang and Tsai [13] considered a three-layer system where the middle sublayer is impermeable but conducting, and of finite thickness. They showed that the system is at its most stable when the partition is located centrally, and the system also becomes more stable as the partition thickness increases or the partition conductivity decreases. The paper by Postelnicu [14] is also of relevance.

Of some interest is the fact that the presence of layering can cause the neutral stability curve to adopt a shape other than that

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Nomenclature

A, B, C, D	constants
c	constant in Eq. (43)
$\underline{d}$	vector
f, g	reduced forms of perturbations
$\mathcal{F}$	the dispersion relation
$\hat{g}$	gravity
H	height of each sublayer
k	disturbance wavenumber
$\hat{k}$	scaled value of k
K	permeability
$\mathcal{M}, \mathcal{N}$	4×4 matrices
N	number of sublayers
p	pressure
Ra	Darcy–Rayleigh number
S	scaled value of Ra
t	time
u	horizontal velocity
$\underline{v}$	vector of coefficients
w	vertical velocity
x	horizontal coordinate
z	vertical coordinate

Greek symbols

α	exponent
β	thermal expansion coefficient
δ	equal to $N^{-1/2}$
ΔT	reference temperature drop
θ	temperature
Θ	disturbance temperature
κ	thermal diffusivity
λ, σ	constants
μ	dynamic viscosity
ρ	density
ψ	streamfunction
Ψ	disturbance streamfunction

Subscripts and superscripts

(b)	basic state
c	critical conditions
j	sublayer index
m	iteration number
$'$	derivative with respect to z
$1, 2, \ldots$	pertaining to a sublayer

with the classical single minimum. McKibbin and O'Sullivan [9] found cases where the neutral curve has a double minimum, and it is often the case that a small change in a system parameter (such as the permeability of one of the sublayers) causes a discontinuous change in the critical wavenumber; this is due to the neutral curve having two minima and the small change in the parameter has simply caused the identity of the mode with the smaller Rayleigh number to swap from one wavenumber to the other. Rees and Riley [15] provided a systematic analysis of two- and three-layer configurations and they traced out the locus in parameter space where such bimodality arises. They also found that it is possible to have three-layer configurations which are trimodal, i.e. that three minima exist each having the same Rayleigh number. Moreover, they also determined regions in parameter space where square-cell convection forms the stable pattern.

In the present paper we will be considering the onset of convection in a porous layer where the layering is brought about by having equally-spaced infinitesimally thin horizontal partitions within the layer. Thus all the sublayers are identical in all of their properties. Such a configuration is an extension of the recent work by Genç and Rees [1] who considered a two-layer system. Much of the analysis we present arises from the dispersion relation which may be calculated by hand fairly easily for two- and three-layer systems, but which may be computed easily for larger numbers of sublayers. It is found that this type of layered system has some unusual properties, namely (i) the neutral curves clump together into groups of N curves when there are N sublayers; (ii) that the curves comprising each group lie within an $O(k^{-1})$ distance of one another when the wavenumber, k, is large; (iii) the critical Darcy–Rayleigh number and wavenumber for the first mode tend towards the respective values 12 and 0 as the number of sublayers increases, which corresponds to the single-layer values when constant heat flux boundary conditions are applied.

2. Governing equations

We consider the onset of convection in a horizontal porous layer which is comprised of a number of identical sublayers which are themselves separated by infinitesimally thin impermeable partitions. Therefore, while fluid may not pass between sublayers, conductive heat transfer is unhindered by the presence of these partitions. A five-sublayer version of the configuration we consider is shown in Fig. 1.

We will assume that the Boussinesq approximation is valid, that the porous medium is homogeneous and isotropic, that the phases are in local thermal equilibrium, and that the fluid motion satisfies Darcy's law with the additional effect of buoyancy. Each sublayer has height, H, which means that a system comprised of N sublayers has height, NH. We will use H as the value against which to nondimensionalise the governing equations, rather than NH; this has the advantage of yielding much easier comparisons between cases which consist of different numbers of sublayers, particularly the classical single-layer Darcy–Bénard problem.

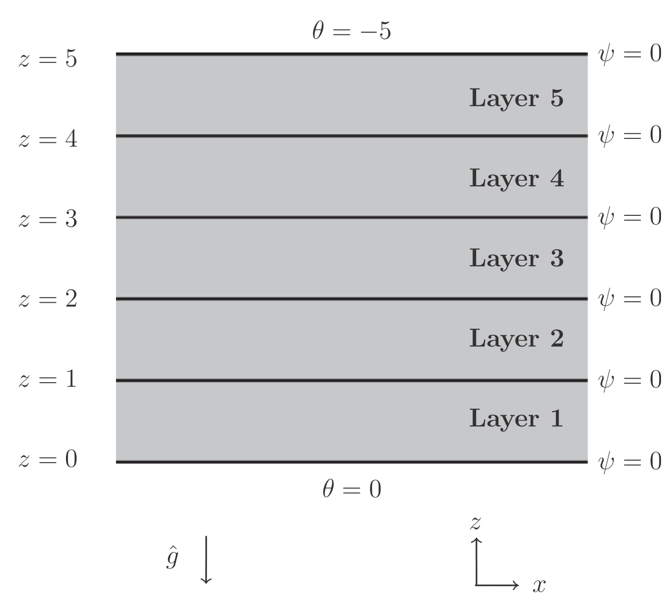

Fig. 1. Depicting a horizontally layered porous medium consisting of five sublayers.