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Turbulent collision rates of arbitrary-density particles

Leonid I. Zaichik [a,*], Olivier Simonin [b], Vladimir M. Alipchenkov [c]

[a] Nuclear Safety Institute of the Russian Academy of Sciences, B. Tulskaya 52, 115191 Moscow, Russia
[b] Institut de Mécanique des Fluides de Toulouse, UMP CNRS/INPT/UPS, Allée du Professeur Camille Soula, 31400 Toulouse, France
[c] Institute for High Temperatures of the Russian Academy of Sciences, Krasnokazarmennaya 17a, 111116 Moscow, Russia

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ABSTRACT

The paper deals with collisions resulting from the interaction between particles (droplets, bubbles) and turbulent eddies of the continuous fluid medium (gas or liquid). A statistical model is developed for predicting the collision rate. This model is valid for arbitrary values of the particle-to-fluid density, the particle inertia parameter, and the ratio between the particle size and the fluid turbulent lengthscale.

1. Introduction

The rate of coagulation due to turbulent collisions of solid particles, liquid droplets or gas bubbles is of importance in many environmental and industrial processes. A large body of theoretical studies of the collision rate induced by turbulence has been performed because of practical significance of the problem. Relatively simple solutions to this problem may be apparently derived only for homogeneous isotropic turbulence. Two analytical solutions are most familiar in the literature, corresponding to the limiting cases of zero-inertia and high-inertia heavy particles. The first solution is valid for fine particles whose response time is much shorter than the Kolmogorov timescale. Fine particles completely follow the velocity fluctuations of the carrier fluid, and their collision rate is determined by interaction with small-scale energy-dissipating turbulent eddies [1]. The second solution pertains to the opposite case of coarse high-inertia particles, whose motion is statistically independent and similar to the chaotic motion of molecules in the kinetic theory of rarefied gases. The collision rate of coarse particles is determined by their interaction with large-scale energy-containing turbulent eddies [2]. The major theoretical difficulties in predicting the turbulence-induced collision rate arise for particles of intermediate size, when the ratios of the particle response time to the turbulence micro- and macroscales are finite ($\tau_k \leqslant \tau_p \leqslant T_L$). In this situation, it is necessary to take into consideration the interactions of particles with all the spectrum of turbulent eddies as well as to account for the correlation of motion of neighbouring particles. More advanced at present are models for predicting the turbulent rate of heavy small particles, when the drag force of all the interfacial forces is only essential (e.g., see [3–6]). The analytical dependence obtained in [7] for heavy small monodisperse (identical) particles describes the contribution of the particle interaction with all the fluid turbulent eddies to the collision rate and is valid over the entire range of particle inertia (from the zero-inertia to the high-inertia limit). In [8], this dependence is generalized to the case of turbulent collisions of bidisperse (different) particles. It is worth nothing that a substantial increase in the collision-coagulation rate makes the effect of particle preferential concentration [5,7–16]. A significant role in the collision-coagulation rate of inertial particles may also play the formation of fold caustics with a multivalued particle velocity field [17–19]; this mechanism can give an extra contribution to the collision rate due to the so-called sling effect [20].

The theory of turbulent collisions of particles suspended in a medium of like density as well as of light particles and bubbles dispersed in a liquid is advanced to a far less extent. In these cases, the turbulence-induced collision rate, as a rule, is determined by analogy with molecular collisions in the kinetic theory of gases, and the relative velocity between particles whose sizes belong to the inertial interval is expressed in terms of turbulence dissipation rate and particle diameter as $(\varepsilon d)^{1/3}$ (e.g., see [21–23]). The present paper deals with a statistical model for the turbulent collision rate, which is valid over the entire range of the particle-to-fluid density. This paper extends the approach used in [7,8] for heavy particles whose size is much less that the Kolmogorov lengthscale to the

* Corresponding author. Tel.: +7 495 955 2394; fax: +7 495 958 1151.
E-mail address: zaichik@ibrae.ac.ru (L.I. Zaichik).

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Nomenclature

A	particle-to-fluid density parameter
a_0	dimensionless acceleration magnitude
d	radius of the collision sphere, $(d_1 + d_2)/2$
d_α	particle diameter
f_u	response coefficient
L	turbulence spatial macroscale
Re_λ	Taylor-scale Reynolds number
S_{ll}	longitudinal structure function
St	Stokes number, τ_{p0}/τ_k
T_L	Lagrangian integral timescale
T_{Lp}	eddy-particle interaction timescale
t	time
u_i	fluid velocity
u'_i	fluctuating fluid velocity
u'^2	fluid velocity variance, $\langle u'_k u'_k\rangle/3$
$\langle u'_k v'_{\alpha k}\rangle$	fluid–particle velocity covariance
$\mathbf{V}_r$	mean velocity between the particulate and fluid phases
$v_{\alpha i}$	particle velocity
$v'_{\alpha i}$	fluctuating particle velocity
v'^2_α	particle velocity variance, $\langle v'_{\alpha k} v'_{\alpha k}\rangle/3$

Greek symbols

β	collision kernel
γ	drift parameter, $\lvert\mathbf{V}_r\rvert/u'$
ε	turbulence dissipation rate
ζ_{in}	two-particle correlation coefficient
η	Kolmogorov lengthscale, $(\nu_f^3/\varepsilon)^{1/4}$
ξ_α	fluid–particle correlation coefficient
ν_f	fluid kinematic viscosity
ρ_f, ρ_p	fluid and particle densities
τ_k	Kolmogorov timescale, $(\nu_f/\varepsilon)^{1/2}$
τ_p	particle response time
τ_{p0}	Stokes particle response time, $\rho_p d^2/18\rho_f \nu_f$
τ_T	Taylor differential timescale
Ω	particle inertia parameter, τ_{p*}/T_{Lp}

Subscripts

α	particle group, 1 or 2
f	fluid
p	particle

case of both arbitrary particle-to-fluid density and arbitrary ratio between the particle size and the fluid turbulent lengthscale. The model being presented describes the collisions caused by the relative velocity between neighboring particles (the so-called turbulent transport effect), but does not account for the nonuniform spatial distribution of particles (the accumulation effect). The effects of sedimentation, hydrodynamic and molecular forces, Brownian motion, and collision efficiency are not taken into consideration as well. The volume fraction of particles is assumed to be low enough for the two-phase system to be treated as a dilute dispersed medium. Moreover, the particles are assumed to be spherical and undeformable and their rotation is not allowed for.

2. Collision kernel

In homogeneous isotropic turbulence, there is a spherical symmetry of the relative motion of two colliding particles. Therefore, the number of collisions of particles belonging to different groups per unit volume per unit time is defined as [5,9]

$$\Sigma_{12} = \beta N_1 N_2, \quad \beta = 2\pi d^2 \langle |w_r(d)|\rangle \Gamma(d), \tag{1}$$

where N_α is the number of particles of group α, $\langle|w_r|\rangle$ is the mean radial relative velocity magnitude of two particles, and Γ is the radial distribution function. Here $\langle|w_r|\rangle$ and Γ quantify, respectively, the relative velocity between neighboring particles (the turbulent transport effect) and the nonuniform particle spatial distribution (the accumulation effect). In the present paper, the accumulation effect is not taken into account, and further $\Gamma(d)$ is assumed to be equal to unity.

The starting point of the model under consideration is the assumption that the one-point joint PDF of fluid and particle velocities as well as the two-point PDF of fluid velocities are the Gaussian distributions (Laviville et al. [24]). These assumptions may derive the following expression for the mean radial relative velocity magnitude of two colliding particles [7,8]:

$$\langle|w_r(d)|\rangle = \left[\frac{2}{\pi}\left(v'^2_1 + v'^2_2 - 2\zeta_{12} v'_1 v'_2\right)\right]^{1/2}, \tag{2}$$

$$\zeta_{12} = \zeta_{in}F(d), \quad \zeta_{in} = \xi_1\xi_2, \quad \xi_\alpha = \frac{\langle u'_k v'_{\alpha k}\rangle}{\langle u'_k u'_k\rangle^{1/2}\langle v'_{\alpha k} v'_{\alpha k}\rangle^{1/2}}.$$

Here ζ_{12} is the correlation coefficient of the radial velocity components of two colliding particles due to their interaction with the fluid turbulence, ζ_{in} is the correlation coefficient of the velocities of two particles due to their inertia, and ξ_α denotes the fluid–particle velocity correlation coefficient that is defined as the ratio of the fluid–particle velocity covariance to their variances. The space longitudinal correlation function $F(d)$ allows for the spatial correlation of the fluid velocities at two points separated by a distance d, which is equal to the spacing between the centers of colliding particles upon their contact. As is seen, the inertia-induced two-particle correlation coefficient, ζ_{in}, is equal to the product of the fluid–particle correlation coefficients, ξ_α, and the total two-particle correlation coefficient, ζ_{12}, is equal to the product of the inertia-induced two-particle correlation coefficient, ζ_{in}, and the longitudinal correlation function, $F(d)$.

In view of (A6), Eqs. (A4) and (A5) yield

$$\langle v'_{\alpha k} v'_{\alpha k}\rangle = \frac{2\Omega_\alpha + 2A^2\Omega_\alpha^2 + z^2}{2\Omega_\alpha + 2\Omega_\alpha^2 + z^2}\langle u'_k u'_k\rangle, \quad \langle u'_k v'_{\alpha k}\rangle = \frac{2\Omega_\alpha + 2A\Omega_\alpha^2 + z^2}{2\Omega_\alpha + 2\Omega_\alpha^2 + z^2}\langle u'_k u'_k\rangle, \tag{3}$$

$$\zeta_{in} = \frac{(2\Omega_1 + 2A\Omega_1^2 + z^2)(2\Omega_2 + 2A\Omega_2^2 + z^2)}{\left[(2\Omega_1 + 2\Omega_1^2 + z^2)(2\Omega_1 + 2A^2\Omega_1^2 + z^2)(2\Omega_2 + 2\Omega_2^2 + z^2)(2\Omega_2 + 2A^2\Omega_2^2 + z^2)\right]^{1/2}}. \tag{4}$$

In the limit of high Reynolds numbers ($z \to 0$ for $Re_\lambda \to \infty$) when neglecting the crossing trajectory effect ($\gamma \to 0$), Eqs. (3) reduce to the well-known relations [25] which correspond to the one-scale exponential autocorrelation function $\Psi_L(\tau) = \exp(-\tau/T_L)$

$$\langle v'_{\alpha k} v'_{\alpha k}\rangle = \frac{1 + A^2\Omega_\alpha}{1 + \Omega_\alpha}\langle u'_k u'_k\rangle, \quad \langle u'_k v'_{\alpha k}\rangle = \frac{1 + A\Omega_\alpha}{1 + \Omega_\alpha}\langle u'_k u'_k\rangle.$$

For a monodisperse system of particles, the inertia-induced two-particle velocity correlation coefficient (4) reduces to

$$\zeta_{in} = \frac{(2\Omega + 2A\Omega^2 + z^2)^2}{(2\Omega + 2\Omega^2 + z^2)(2\Omega + 2A^2\Omega^2 + z^2)}. \tag{5}$$

Fig. 1 shows ζ_{in} as a function of the particle inertia parameter Ω at high Reynolds numbers when $z = 0$. For heavy particles dispersed in a light continuous medium ($\rho_p/\rho_f = \infty$, $A = 0$), a monotonous decrease in ζ_{in} takes place from unity to zero as Ω increases. However, in the case of finite values of the particle-to-fluid density ρ_p/ρ_f when $A > 0$, the dependence $\zeta_{in}(\Omega)$ has a minimum. It is worth nothing that, in this case, the inertia-induced two-particle velocity