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Heat transfer in a second grade fluid through a porous medium from a permeable stretching sheet with non-uniform heat source/sink

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ABSTRACT

In the present article an analysis is carried out to study the boundary layer flow and heat transfer characteristics of a second grade, non-Newtonian fluid through a porous medium. The stretching sheet is assumed to be permeable so that suction effects come into play. The effects of viscous dissipation, non-uniform heat source/sink on heat transfer are addressed. The basic boundary layer equations for momentum and heat transfer, which are non-linear partial differential equations, are converted into non-linear ordinary differential equations by means of similarity transformation. Analytical solutions are obtained for the resulting boundary value problems. The effects of viscous dissipation and non-uniform heat source/sink, Prandtl number, Eckert number and suction/injection on heat transfer are shown in several plots for two different heating processes (CST and PST cases). Dimensionless surface temperature gradient is tabulated for various values of the governing the parameters.

1. Introduction

Boundary layer behavior over a moving continuous solid surface is an important type of flow occurring in several engineering processes. The heat transfer due to a continuously moving stretching surface through an ambient fluid is one of the thrust areas of current research. Such investigations find their application over a broad spectrum of science and engineering disciplines, especially in the field of chemical engineering. Many chemical engineering processes like metallurgical process, polymer extrusion process involves cooling of a molten liquid being stretched into a cooling system. The fluid mechanical properties desired for an outcome of such a process would mainly depend on two aspects, one is the cooling liquid used and the other is the rate of stretching. Fluids of non-Newtonian characteristics which are electrically conducting can be opted as a cooling liquid as their flow can be regulated by external magnetic field which regulates heat transfer to some extent. Rate of stretching is very important as rapid stretching results in sudden solidification thereby destroying the properties expected for the outcome.

The problem mentioned here is a fundamental one and frequently arises in many practical situations such as polymer extrusion process. It is also encountered in other process like drawing, oil reservoirs, geothermal engineering, annealing and tinning of copper wires, continuous stretching, rolling and manufacturing of plastic films and artificial fibers, heat treated materials traveling on conveyer belts, glass blowing, crystal growing, paper production and so on.

Sakiadis [1,2] initiated the study of the boundary layer flow over a continuous solid surface moving with constant speed. The boundary layer problem considered by Sakiadis differ form the Classical Boundary layer problem addressed by Blasius mainly due to the entrainment of the ambient fluid. Here the surface is assumed to be inextensible ($u_w = 0$) where as most of the physical situations concern with extensible surfaces ($u_w = bx$) moving in a cooling liquid. Crane [3] is the first to consider the boundary layer behavior over an extensible surface where he assumes the velocity of the surface to vary linearly with the distance form the slit.

Works of Carragher and Crane [4], Vleggaar [5] and Gupta and Gupta [6] address flow and heat transfer from a stretching sheet to a Newtonian fluid. Chen and Char [7] have extended the works of Gupta and Gupta [6] to that of non-isothermal stretching sheet. These works concern only Newtonian fluids. In reality most liquids are non-Newtonian in nature, which are abundantly used in many industrial and engineering applications. To name a few industrially important fluid are viscoelastic fluids, second grade fluids, power law fluids and others. In view of this, the study of boundary layer behavior has been further channelised to non-Newtonian fluids.

Vajravelu and Rollins [8] have investigated hydro magnetic flow of a second grade fluid over a stretching sheet. Thermodynamics stability analysis of fluids of second grade is carried out by of Dunn

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Nomenclature

A	prescribed constant
c_p	specific heat at constant pressure
λ_1	second grade parameter
λ_2	porosity parameter
R	dimensionless suction/blowing parameter
k	thermal conductivity
α	thermal diffusivity
α_1	material constant
u, v	velocity components along x and y direction
T	fluid temperature of the moving sheet
T_w	wall temperature
F, M	confluent hyper geometric function
Pr	Prandtl number
Ec	Eckert number
T_∞	temperature far away from the plate
k'	permeability of the porous medium
q'''	rate of internal heat generation/absorption
A^*	space-dependent internal heat generation/absorption
B^*	temperature-dependent internal heat generation/absorption
c	stretching rate
τ_w	wall shearing stress
C_f	skin-friction coefficient.

Greek symbols

θ	dimensionless temperature
η	dimensionless space variable
ν	kinematic viscosity
ρ	density
μ	coefficient of viscosity

Subscripts

w	properties at the plate
∞	free stream condition

Superscript

$'$	differentiation with respect to η

and Fosdick [9]. Cortell [10] considered flow and heat transfer of an electrically conducting fluid of second grade over a stretching sheet subject to suction and a transverse magnetic field. There are also investigations related to viscoelastic fluids, see Refs. [11–18].

The above investigators restrict their analysis to flow behavior in non-porous media. The rate of cooling can be controlled if strips are drawn through porous media. In view of this, the study of non-Newtonian fluid flow through porous media has gained importance in recent years; see references [19–22]. Recently, Subhas Abel et al. [23] investigated the effect of variable viscosity on viscoelastic fluid flow and heat transfer in a porous medium over a non-isothermal stretching sheet.

The effect of heat source/sink is very important in cooling processes. Vajravelu and Rollins [24], Vajravelu and Nayfeh [25] have studied the effect of uniform heat source/sink on heat transfer from a stretching sheet into a cooling liquid. Abo-Eldahab and El-Aziz [26] investigated heat transfer considering non-uniform heat source/sink. But these studies are confined to flow and heat transfer in Newtonian fluids. Cortell [27], Abel et al. [28], Abel and Mahesha [29] have considered the effects of non-uniform heat source but in case of viscoelastic fluid. In all above literatures [24–29] the effect of non-uniform heat source is considered in non-porous medium.

Hayat et al. [34,35] studied heat and mass transfer characteristics on steady and unsteady flow of second grade fluid and fluid film. Sajid et al. [36] also investigated the unsteady flow of second grade fluid over a stretching shett. Asghar et al. [37] studied the unsteady coutte flow of second grade fluid with variable material properties.

All works [34–37] neglected the effects of non-uniform heat source and viscous dissipation effects on heat transfer.

Though the works of Seddeek [17], Abel and Veena [19], Gupta and Sridhar [20], Prasad et al. [21,22], Abel et al. [23], are flow through a porous medium but they neglected the contribution of non-uniform heat source as well as the effects of new model of viscous dissipation (New model of viscous dissipation as suggested by Hadrami et al. [30,31], which was reported by Pantokratoras [32], while commenting on work of Aouadi [33]). Also Pantokratoras [32] reported that contribution of Hadrami model of viscous dissipation in energy equation is still open in the literature, hence in the present problem we investigated the combined effects of new model of viscous dissipation, non-uniform heat source on heat transfer characteristics, of second grade fluid over a permeable stretching sheet.

Heat transfer characteristics are examined for different kinds of boundary heating, namely constant surface temperature (CST case) and prescribed surface temperature (PST case).

2. Mathematical formulation and solution

Consider a steady, laminar and two-dimensional flow of an incompressible, second grade fluid through a porous medium past a flat sheet coinciding with the plane $y = 0$ and the flow being confined to $y > 0$.

The flow is generated, due to stretching of the sheet, caused by the simultaneous application of two equal and opposite forces along the x-axis. Keeping the origin fixed, the sheet is then stretched with a speed varying linearly with the distance from the slit. We take x-axis along the surface, y-axis being normal to it and u and v are the fluid tangential velocity and normal velocity, respectively.

The equations governing the problem under consideration are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left\{\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y} + v\frac{\partial^3 u}{\partial y^3}\right\} - \frac{\nu}{k'}u \tag{2}$$

where υ is the kinematic viscosity, ρ is the density, α_1 is the material constant and k' is the permeability coefficient of porous medium.

The boundary conditions for the velocity field are of the form

$$\begin{aligned} &u = cx, \quad v = -v_0 \quad \text{at} \quad y = 0 \\ &u \to 0 \quad \text{as} \quad y \to \infty \end{aligned} \tag{3}$$

where c is the stretching rate.

We define the following new variables

$$u = cxf'(\eta), \quad v = -(c\upsilon)^{\frac{1}{2}}f(\eta), \quad \eta = \left(\frac{\upsilon}{c}\right)^{-\frac{1}{2}}y \tag{4}$$