International Journal of Heat and Mass Transfer 53 (2010) 2193–2205

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Multi-parameter model reduction in multi-scale convective systems

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ARTICLE INFO

Article history:
Received 3 April 2009
Received in revised form 5 November 2009
Accepted 5 November 2009

Keywords:
Multi-scale thermal-fluid systems
Reduced order modeling
Proper orthogonal decomposition
Galerkin projection
Data center

ABSTRACT

A new Proper Orthogonal Decomposition (POD) based reduced order modeling approach for temperature field calculation in multi-scale convective systems is presented. Using POD modes for the entire domain, the energy equation is solved only at the most important scales via Galerkin projection to have an efficient and accurate enough model for design. Comparing with Computational Fluid Dynamics/Heat Transfer simulation shows that the method can predict the temperature field at the rack scale of a sample air-cooled data center with the average error norm of ~6% for different sets of design parameters, while it can be up to ~250 times faster.

1. Multi-scale systems simulation for thermal design

Full-field simulations are currently used to predict the flow and temperature fields inside multi-scale thermal-fluid systems. Using CFD/HT for multi-scale systems design is time-consuming and costly, and is not practical for iterative optimization based methods. An optimal thermal design is possible only if general simulation methods are available for rapid and accurate modeling of the flow and temperature fields within multi-scale systems. In this regard, compact or reduced models which could run significantly faster than CFD/HT models, while incorporating important scale parameters with sufficient fidelity are essential.

A representative example of a multi-scale turbulent convective system in need of reduced modeling is a data center. Data centers, as shown in Fig. 1, are computing infrastructure facilities that house arrays of electronic racks containing high power dissipation data processing and storage equipment whose temperatures must be maintained within allowable limits. As seen in Fig. 1, data centers have a multi-scale nature. The typical approach currently used for thermal management of data centers utilizes computer room air-conditioning (CRAC or AC) units that deliver cold air to the racks arranged in alternate cold/hot aisles through perforated tiles placed over an under-floor plenum, see Fig. 2. Meta-modeling techniques [1] can be used to extract the dominant characteristics of such systems, trading a degree of accuracy for computational speed. Simulation methods based on machine learning methodology to predict the air temperature at discrete points, such as server inlets/outlets for a new heat load distribution among the data center racks have also been explored [2–5]. These methods require a large number of data points for interpolation and usually need a long calibration for each data center of interest before they can be used for simulation.

In this paper, a new Proper Orthogonal Decomposition (POD) based reduced order modeling of temperature field in multi-scale convective systems is presented to efficiently simulate the effect of the design parameters. The flow field is considered known, in order to focus on the temperature predictions. In prior work [6,9] the POD method has been successfully applied to the development of reduced order flow models, and if needed could be used in conjunction with the present study. The approach is applicable for systems where the buoyancy effects are negligible and the temperature field at selected scales drives the thermal design decision. The energy equation is solved only at these dominant scales via system POD modes and Galerkin projection to obtain a more accurate zoomed prediction at these scales, instead of the entire domain. The effects of the phenomena at other scales are modeled through simple energy balance equations and known heat flux and temperature matching, as well as appropriate matching conditions at the scale interfaces.

In Section 2, the basic POD technique and its enhancement to simulate the temperature field in multi-scale thermal-fluid systems are explained. In Section 3, the method is applied to an air-cooled data center cell with 5 design variables. The accuracy and efficiency of the POD generated temperature field for different sets of involved design parameters are examined through comparison with CFD/HT results. The results are reviewed and discussed in Section 4. A novel feature is the use of POD modes and Galerkin projection for solving the governing turbulent convection equation in a complex multi-scale system. To the best of the authors' knowl-

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doi:10.1016/j.ijheatmasstransfer.2009.12.013

Nomenclature

a,b	modal weight coefficient
c_p	specific heat
k_{eff}	effective thermal conductivity
k_{th}	thermal conductivity
n	number of observations
m	number of retained modes
P	pressure
Q	heat generation
q	volumetric heat generation
T	temperature
u	velocity field
V	average velocity on a surface

Greek symbols

λ	eigenvalue
ρ	density
ψ	POD mode

Subscripts

obs	observation
cond	conduction

Superscripts

$*$	transpose
$+$	pseudo-inverse

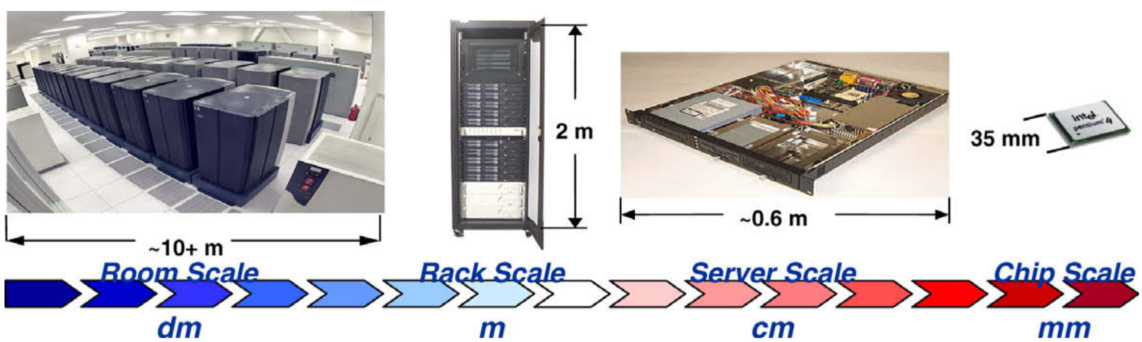

Fig. 1. Typical data center and its multi-scale nature.

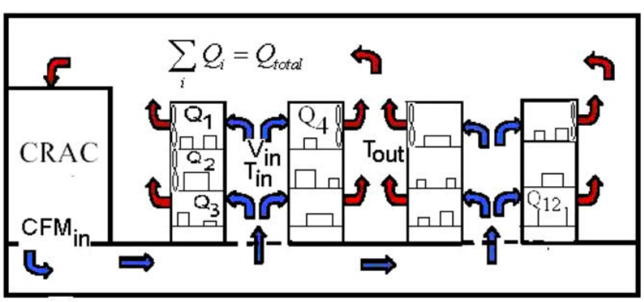

Fig. 2. Typical air cooling system in data centers.

edge, this paper is the first attempt to develop multi-parameter reduced order thermal modeling of multi-scale convective systems.

2. Reduced order thermal modeling in multi-scale systems

In Section 2.1, the basic POD technique is explained briefly, while more details can be found in [6–9]. In Section 2.2, the new POD based method to simulate the temperature field is explained.

2.1. Proper orthogonal decomposition

The Proper Orthogonal Decomposition (POD), also known as the Karhunen-Loeve decomposition, is a model reduction technique that expands a set of data on empirically determined basis functions for modal decomposition. It can be used to numerically predict the temperature field more rapidly than full-field simulations. The temperature field is expanded into basis functions or POD modes:

$$T = T_0 + \sum_{i=1}^{m} b_i \psi_i \quad (1)$$

The POD modes of a thermal system, ψ_i, can be calculated from observations, that are obtained by CFD /HT simulations or detailed experimental measurements by changing the design parameters of the system n-times. In Eq. (1), m is the number of retained POD modes in the decomposition which can be 1 up to $n-1$. Using the method of snapshots, each POD mode can be expressed as a linear combination of the linearly independent observations [7]:

$$\psi_i = \sum_{k=1}^{n} a_k (T_{obs,k} - T_0) \quad (2)$$

where T_{obs} is a matrix of which each column, $T_{obs,i}$, includes a complete temperature field data from an observation. An element of the reference temperature field, T_0, in Eqs. (1) and (2) is usually considered as the average of the all observed data for a field point. The weight coefficients, a_k, in Eq. (2) are obtained by solving the following $n \times n$ eigenvalue problem:

$$\sum_{k=1}^{n} R(i,k) a_k = \lambda a_i; \quad i = 1, \ldots, n \quad (3)$$

where $R = (T_{obs} - T_0)^* \otimes (T_{obs} - T_0)/n$ and n is the number of observations [6–9]. For a given set of observations, n eigenvalues, λ_i, and their relevant eigenvectors are obtained from Eq. (3). Each eigenvector includes the weight coefficients, a_k, of the relative POD mode in Eq. (2), so n POD modes are finally calculated. The energy captured by each POD mode in the system is proportional to the relevant eigenvalue. The eigenvalues are sorted in a descending order, so the first POD modes in Eq. (1) capture larger energy compared with the later modes.

The POD has several properties that make it well suited for turbulent flows. First, the empirical determination of the basis func-