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Taylor dispersion in a two-zone packed tube

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ABSTRACT

Heterogeneity of media distribution in a packed tube plays an important role on scalar dispersion by enhancing the transverse non-uniformity of the flow velocity. Presented in this paper is a theoretical analysis for an instantaneous release of scalar substance into a fully developed flow through a long tube of two zones distinctively packed with porous media. The velocity distribution of the flow through the tube is derived, with the known solution for a single zone tube flow included as a special case. Mei's perturbation analysis for scalar dispersion in a single phase fluid flow is rigorously generalized for the two-zone case of a tube flow to develop a dispersion model by averaging the concentration transport equation. Corresponding dispersivity is analytically determined, and Taylor's well-known result for a single-zone tube flow is recovered by setting corresponding parameters as unity. The enhancement of the dispersion by the heterogeneity and the dependence of the enhanced dispersion on the tube radius are illustrated and characterized by relevant dimensionless parameters.

1. Introduction

Taylor dispersion [1] refers to the process that scalar substances spread longitudinally by a diffusion-like process in a confined shearing flow under combined action of lateral diffusion and flow speed non-uniformity, as intrinsic in a variety of flows including those in rivers, estuaries, blood vessels and lung tracheas [2–7].

The effect of Taylor dispersion in porous media can be essential in a variety of practical applications such as the extraction of energy from geothermal regions, biochemical separation or purification of mixtures, biomechanical practices of cartilage in synovial joints [8], improving efficiency of conventional oil recovery by applying Enhanced Oil Recovery processes [9], and environmental issues including environmental risk assessment, ecological restoration and wastewater treatment engineering associated with wetlands [10,11].

Flow and dispersion through packed glass beads have been intensively measured by nuclear magnetic resonance (NMR) spectrometer experiments and conventional column breakthrough experiments [12]. Maier et al. simulated the enhanced dispersion in sphere-packed tube in the pore scale [13] with fluid velocity field calculated by lattice-Boltzmann method [14] and tracer motion captured by a random-walk particle-tracking algorithm. The time scale needed for the dispersion to attain its asymptotic rate has been characterized as the square of the packed tube radius over the bulk transverse dispersion rate [15]. Experiments with different tube inner diameters were conducted to confirm the dependence of dispersivity on the tube radius [16].

Different from the approaches focused on the pore-scale processes, Chen and Zeng [17] analytically studied Taylor dispersion in a packed tube at the holistic scale, on the basis of phase average. Complex as transport in porous media, it is hard to figure out the details of the real flow and concentration transport at the pore scale associated with the unknown interface between fluid in the irregular vicinity and solid frame. By the operation of phase average, discontinuity caused by the interface of fluid and solid frame could be smeared out and the resulted superficial field [18,19] is a continuous distribution in the entire domain of concern. Thus the term Taylor dispersion refers to the macroscopic phenomenon due to the existence of velocity gradient in the superficial flow. The superficial analysis has been extended to explore concentration dispersion in wetland flows [2,20–24].

While all the experimental, numerical and analytical researches have been performed on the dispersion in a uniformly packed tube, the effect of inhomogeneities in porous media remains to be assessed.

Heterogeneity of media distribution can play an important role on scalar dispersion by enhancing the non-uniformity of the flow velocity. For example, for scalar dispersion associated with blood flows in arteries the condition of atherosclerosis can be illustrated by the model of a two-zone packed tube. Although under certain conditions blood displays some viscoelastic properties, it is commonly believed that the influence of the non-Newtonian property is meager in large arteries where the shear rate is high [25]. Atherosclerosis is a disease caused by different factors, among which the transport and accumulation of atherogenic low-density

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Nomenclature

a_i	radius of zone 1 ($i = 1$) and thickness of zone 2 ($i = 2$)
b	ratio of radius of zone 1 to that of the entire tube
C	concentration
C_i	concentration in zone i ($i = 1,2$)
$C_i^{(j)}$	jth order concentration ($j = 0,1,2$) in zone i ($i = 1,2$)
C_b	an integral constant
C^*	dimensionless concentration
D	enhancement of Taylor dispersivity
D_R	radius-dependent dispersivity
D_s	Taylor dispersivity for the single-zone tube case
D_T	Taylor dispersivity
F	shear factor
F_i	shear factor in zone i ($i = 1,2$)
f	any property
f_i	any property in zone i ($i = 1,2$)
$\mathbf{K}$	concentration dispersivity tensor
K_i	concentration dispersivity in zone i ($i = 1,2$)
$\mathbf{L}$	momentum dispersivity tensor
L_i	momentum dispersivity in zone i ($i = 1,2$)
M	square root of the effective viscosity of the porous media in zone 2 over that in zone 1
N	square root of the viscous friction of the porous media in zone 2 over that in zone 1
P	pressure
Pe_i	Peclet number in zone i ($i = 1,2$)
Q	mass
R	radius of the tube
r	radial coordinate
r_i	radial coordinate for zone i ($i = 1,2$)
T_i	convection time scale ($i = 1$) and diffusion time scale ($i = 2$)
t	time
t_i	different time coordinate for the expansion ($i = 0,1$)
t^*	dimensionless time
$\mathbf{U}$	velocity vector
u_i	longitudinal velocity in zone i ($i = 1,2$)
u_c	characteristic velocity
u_B	dimensionless velocity at the interface of two zones
x	longitudinal coordinate
x_i	longitudinal coordinate for zone i ($i = 1,2$)
X	characteristic length of the concentration cloud

Greek symbols

α	dimensionless parameter
δ	Dirac delta function
ε	small parameter for the multiple scale expansion
ζ	dimensionless longitudinally moving coordinate
η	dimensionless radial coordinate
κ	tortuosity
λ	concentration diffusivity
μ	dynamic viscosity
ξ	dimensionless longitudinal coordinate
ρ	density
τ	dimensionless time
φ	porosity
φ_i	porosity in zone i ($i = 1,2$)
Ψ	dimensionless longitudinal velocity
Ψ_i	dimensionless longitudinal velocity in zone i ($i = 1,2$)
ψ_i	dimensionless longitudinal velocity in zone i ($i = 1,2$), nondimensionalized by the averaged velocity
$\psi_{\varphi i}$	dividing ψ_i by φ_i ($i = 1,2$)
$\psi'_{\varphi i}$	dimensionless velocity deviation in zone i ($i = 1,2$)

lipoproteins (LDL) from flowing blood to the wall of an artery are universally acknowledged [25–27]. The scalar transport in the artery of both single-zone and two-zones is of considerable interest.

For scalar transport in a two-zone packed tube, presented in this work is an analytical study on the flow profile and the longitudinal evolution of the cross-sectional mean concentration under Taylor dispersion. The specific objectives of this paper are: (I) to give a velocity distribution of superficial flow through the two-zone tube; (II) to determine the Taylor dispersivity in the two-zone tube by perturbation analysis; (III) to analyze the enhancement of the dispersion by both the global parameter and relative parameters; (IV) to discuss the dependence of dispersivity on the tube radius in the two-zone packed tube and (V) to illustrate the variation of dimensionless concentration with different parameters.

2. Momentum and concentration transport

Once there is an instantaneous release of scalar substance into a flow through a long tube, the evolution of the concentration cloud is characterized by two stages on the whole. At the very beginning of the release, there is a large longitudinal concentration gradient across the tube section, and the radial concentration difference caused by the radial variation of the longitudinal velocity cannot be balanced by the radial diffusion driven by the radial concentration gradient. The cross-sectional mean concentration thus forms a longitudinal skewed distribution. As time goes by, the longitudinal concentration gradient is reduced, the radial concentration difference gradually turns to be balanced by the radial diffusion, then the longitudinal distribution of the cross-sectional mean concentration tends to a normal distribution. Under such circumstances, the centroid of the cloud moves at the cross-sectional mean velocity of the flow, and the cloud disperses longitudinally by a virtual diffusion coefficient [7,28]. The latter stage of the evolution is known as the Taylor dispersion.

For a typical flow in the packed tube, basic equations for momentum and mass transport can be adopted generally at the phase average scale as [2,18,29]

$$\rho\left(\frac{\partial \mathbf{U}}{\partial t} + \nabla\cdot\frac{\mathbf{UU}}{\varphi}\right) = -\nabla P - \mu F\mathbf{U} + \kappa\mu\nabla^2\mathbf{U} + \kappa\nabla\cdot(\mathbf{L}\cdot\nabla\mathbf{U}), \tag{1}$$

$$\varphi\frac{\partial C}{\partial t} + \nabla\cdot(\mathbf{U}C) = \nabla\cdot(\kappa\lambda\varphi\nabla C) + \kappa\nabla\cdot(\mathbf{K}\cdot\nabla C). \tag{2}$$

Like viscosity for momentum transfer and diffusivity for concentration transfer valid for the description of the single phase fluid flow at the microscopic passage scale, momentum and concentration dispersivities are properties valid for the description of superficial flow at the phase-average scale out of the operation of phase average to smear out the discontinuity between the two phases of the ambient fluid and the porous media. The expression of the momentum equation for the superficial flow through porous media is out of a combination of the Navier–Stokes equation for single phase fluid flows and Darcy's law for sweeping flows in porous media plus a term of second-order derivative to account for momentum dispersion. Similarly, the equation for concentration transport is out of a combination of an advective–diffusive equation and a concentration dispersion law.

Consider scalar transport in a fully-developed unidirectional flow through a two-zone packed tube with length much longer than the radius $R = a_1 + a_2$, in a Cylindrical coordinate system with longitudinal x-axis aligned with the flow direction, radial r-axis,