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Application of residual correction method to laser heating process

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ABSTRACT

In this study, the residual correction method is employed to predict the temperature distribution during laser heating process. The energy transfer induced by laser irradiation in the solid is described by Fourier's law of conduction with an energy source modeled by Beer's law. The approximate solution of temperature field is obtained by residual correction. Using the residual correction method, the precise average value of upper and lower approximate solutions is obtained and the error range between the analytical solution and the numerical solution can be analyzed. The results reveal that this method is an effective numerical method with satisfactory accuracy.

1. Introduction

The high-power laser beam has many important industrial and medical applications which include drilling, welding, heating, non-contact measurement, sensing and surgical treatment, etc. Laser heating can be used to heat a solid substrate of different material by regulating its process parameters. The physical model of laser radiation absorption was introduced by Ready [1] in 1965, where the analytical solution for a high-power pulse laser heating process of specified shapes absorbed at opaque surfaces was obtained. Their results are also in agreement with the experimental data. Diniz Neto and Lima [2] developed a numerical procedure to investigate the transient behavior of the temperature distribution in a solid heated by short powerful laser pulses. They used the finite difference method to compute the temperature distributions or any desired thermal profiles in any space or time variables. The results were presented and discussed from the point of view of an application of the model to metals heated with a pulsed Nd-YAG laser. Rozzi et al. [3] presented experimental and three-dimension numerical predictions for laser heating process to investigate the thermal response of a rotating silicon nitride workpiece heated by a translating CO_2 laser. The results appraise the probability of the LAM (laser-assisted machining) process and provided a good understanding of laser heating phenomena. In 2001, Yilbas and Kalyon [4] investigated the temperature distribution due to repetitive laser pulse heating with a convective boundary condition at the surface. They utilized Laplace transformation method to obtain the analytical solution for the heat transfer equation. The effects of the pulse parameter and the Biot number on the resulting temperature profiles during repetitive laser pulse heating are examined. The effect of Biot number on surface temperature profiles becomes obvious for $Bi \geqslant 2 \times 10^{-2}$. Besides, the results revealed that the temperature difference between the first and second peak temperatures in the temperature curve is never zero for all laser pulse parameter and Biot number. They concluded that a constant surface temperature is improbable to achieved using repetitive laser pulse heating. He et al. [5] employed the Laplace transform to obtain an analytical solution for the temperature field with convective boundary condition. Moreover, the experiment data obtained by the thermo-camera are used to validate the analytical solution. Their results revealed the influences of Biot number and dimensionless energy absorption at surface in temperature profiles. Peng and Chen [6] utilized the differential transformation method to predict the laser heating problem. The influence of convective boundary and dimensionless energy absorption at the surface are examined. This study provided an effective and efficient method to analyze the heat transfer problem. The analytical solution for repetitive laser pulse heating of a solid substrate was carried out by Kalyon and Yilbas [7] in 2006. The Laplace transformation method was utilized to obtain a closed form solution for the temperature rise inside the material. Their results demonstrated that the rise in temperature due to a single pulse with double intensity is higher than that corresponding to consecutive two pulses without having the cooling period between them. Lo [8] investigated the heat transfer in a thin film exposed to ultrashort-pulsed laser by using the hybrid differential transform/finite-difference method. The governing equations were transformed from the time domain into the spectrum domain using the differential transform method and then discretized in the space domain by the finite-difference method. Then, a recursive procedure was employed to solve the transformed equations and obtain the numerical solutions. The study indicated that although the electron/lattice temperature is at equilibrium at only a couple hundred degrees,

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Nomenclature

Bi	Biot number
$E_{\max}$	the maximum possible error
h	convective heat transfer coefficient (W/m^{2}K)
I_0	laser peak power intensity (W)
k	conductivity (W/mK)
L	linear operator
M	dimensionless energy absorption at surface
N	nonlinear operator
N_i	number of grids
$\dot{q}$	heat source per unit volume
r_f	reflection coefficient
r, x, z	distance (m)
r^*, x^*, z^*	dimensionless distance
t	time (s)
T	temperature (K)
T^*	dimensionless temperature
T_e	temperature at time t (K)
T_o	initial temperature (K)
Greek symbols	
α	thermal diffusivity (m^2/s)
β	laser pulse parameter (1/s)
γ	laser pulse parameter (1/s)
δ	absorption coefficient (1/m)
τ	dimensionless time

the maximum electron temperature can reach as high as several thousand degrees. The laser heating of steel and formation of a melt pool in the laser-heated surface were studied numerically by Shuja and Yilbas [9]. In the study, the enthalpy-porosity method was used to analyze the phase change problem. They predicted the temperature distribution and the velocity field in the melt layer. Tseng and Tsai [10] used the dual-phase-lag model to analyze the microscale heat transfer in metallic films. The diffusion and the CV wave models were also analyzed by using different values of the phase lags in the DPL model. Results showed that the dual-phase-lag model is more accurate by comparing with experiments.

In order to realize the effect of laser heating process, the mathematical model must be constructed. It usually combines with nonlinear differential equations and boundary conditions. In the real world, it is difficult to find the exact solution of nonlinear differential equation. It is only possible to utilize some numerical schemes to find out the approximate solution and the error range. residual correction method is one of them.

The past researches show that error between approximate solution and exact solution usually decreases with an increasing grid points or numbers of approximate function. These methods require more memory space and calculation time. However, it is still unable to determine the accuracy of approximate solution completely. Hence, another theory based on maximum principle of differential equations problem (Protter and Weinberger [11]) is utilized to find the upper and lower approximate solutions and to estimate the error range between approximate solution and exact solution.

However, this kind of method includes a mathematical programming problem of inequalities. It requires complex and heavy load of calculation. In recent years, some scholars have made an effort to simplify the calculating procedure. Wang [12,13], Cheng et al. [14] and Wang [15] utilized the spline approximation to discretize the differential equations into the mathematical programming problems. Then, based on the residual correction concept, the inequality constraint mathematical programming problems can be converted into the simple iterative equations. The results showed that the efficiency of obtaining solutions is raised significantly. Tang et al. [16] extended the previous studies. They applied the finite difference to discretize the equation, converting the differential equation into the mathematical programming problem. And then incorporating the residual correction method to obtain the upper and lower approximate solutions. Their study showed that the methodology of incorporating the residual correction method into the nonlinear iterative procedure of the finite difference can make it easier and faster to obtain the approximation solutions.

In this study, a technique called the residual correction method based on maximum principles in differential equations is utilized to find out the upper and lower approximate solution of laser heating process. The influences of convective boundary and laser pulse parameters are examined.

2. Maximum principles for differential equations

The concept of maximum principle is utilized to establish the residual of differential equations to obtain the upper and lower approximate solutions. The first, suppose that the differential equation in the form as below:

$$R_{\tilde{\theta}}(x) = F(x, \tilde{u}, \tilde{u}_x, \tilde{u}_{xx}) - f(x) \quad \text{in}\, D \tag{1}$$

boundary

$$R_{\tilde{\theta}}(x) = g(x) - \tilde{\theta}(x) \quad \text{on}\, \partial D \tag{2}$$

where $R_{\tilde{\theta}}(x)$ is known as the residual of the differential equation. On assumption that the approximate solutions have definition in the calculation domain and are continuous till second derivatives, if

$$\frac{\partial R}{\partial \theta} \le 0 \quad \text{in}\, D \tag{3}$$

Then, when the following equation holds:

$$R_{\breve{\theta}}(x) \ge R_{\theta}(x) = 0 \ge R_{\widehat{\theta}}(x) \quad \text{on}\, D \cup \partial D \tag{4}$$

The approximate solutions will have the following relation with the exact solution:

$$\breve{\theta}(x) \le \theta(x) \le \widehat{\theta}(x) \quad \text{on}\, D \cup \partial D \tag{5}$$

where $\breve{\theta}(x)$ and $\widehat{\theta}(x)$ are known as the lower and upper approximate solutions of the exact solution $\theta(x)$, respectively. And differential equation with such relations is considered as monotonic.

3. Residual correction steps

Using the finite difference method to discretize and reformulate the residual relation into the following expression:

$$R_{r,i,j,k}(t,x,y,z) = -\left(L[\theta]^{n+1}_{r,i,j,k} + N[\theta]^{n}_{r,i,j,k}\right) + f_{r,i,j,k} \tag{6}$$

where L is the linear operator and N is the nonlinear operator, the superscript n is the iterative times, and the subscript r, i, j, k is the serial number of the grid points after discretizing.

Then, transfer the expression into an iterative equation with residual correction to avoid complex calculations:

$$-\left(L[\theta]^{n+1}_{r,i,j,k} + N[\theta]^{n}_{r,i,j,k}\right) + f_{r,i,j,k} = \begin{matrix}\max \\ \min\end{matrix} \left(\Delta R^{n}_{r,i,j,k}(t,x,y,z)\right) \tag{7}$$