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Characteristics of heating scheme and mass transfer on the peristaltic flow for an Eyring–Powell fluid in an endoscope

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ABSTRACT

In this article Eyring–Powell peristaltic fluid flow with heat and mass transfer analysis have been investigated. New fluid model have been presented in peristaltic literature. The governing equations for proposed Eyring–Powell fluid model are derived in cylindrical coordinates both in fixed and moving frame of reference. Complex system of equations have been simplified using long wavelength and low Reynolds number approximation. The momentum and heat/mass transfer balance equations are solved analytically and numerically by employing perturbation method and shooting technique. Graphical results have been discussed for pressure rise, frictional forces, temperature and concentration profile. Comparison of perturbation and numerical solutions have been presented through table and figures. Five different waves forms have been considered for analysis. Trapping phenomena have been presented for different wave forms.

1. Introduction

The study of non-Newtonian fluid flows has gained much attention by the researchers because of its applications in biology, physiology, technology and industry. In addition, the effects of heat and mass transfer in non-Newtonian fluid [1,2] have great importance in engineering applications like the thermal design of industrial equipment, food stuffs, or slurries etc. Moreover Navier Stokes theory become insufficient when there are some complex rheological fluids. So there is need to study the non-Newtonian fluids [3,4]. Since not a single model exhibits all the properties of fluids, therefore, many non-Newtonian models have been presented by various authors [5–7]. Erying Powell fluid model is one of the non-Newtonian fluid which was presented by Powell and Erying in 1944. This is complex model but up to now not a single author have used this model in peristalltic flow problems in an endoscope.

Peristalsis is a mechanism to pump the fluid by means of a moving contraction on the tubes or channels walls. Such mechanisms mostly occurs in many biological and biomedical systems. In physiology, these play an important role in various systems such as urine transport from kidneys to bladder through the ureter, chyme movement in the gastrointestinal tract, transport of spermatozoa in the ductus afferents of the male reproductive tracts, movements of ovum in the female fallopian tube and circulation of blood in the small blood vessels. In the past few decades, several theoretical and experimental investigations have been made just to understand peristalsis in different situations. The literature on this topic is quite extensive. Mention may be made to some recent studies include in Refs. [8–10] in the field for Newtonian and non-Newtonian fluids. Heat and mass transfer effects on the peristaltic flows in tubes, endoscope, and channels [11–15] are very important because of its practical engineering applications, such as food processing and blood pumps in heart lungs machines.

Eyring–Powell fluid model [16], a complex mathematical model developed by Powell and Eyring in 1944. To the best of our knowledge no investigation have been done to study the Eyring–Powell fluid model in peristaltic literature, therefore to fill this gape present study investigate the importances of peristaltic flow of Eyring–Powell fluid in an endoscope. The governing equations for Eyring–Powell fluid model are formulated considering cylindrical coordinates system. The equations are simplified using the assumptions of long wave length and low Reynold's number approximation. The simplified non-linear differential equations are then solved analytically and numerically by perturbation and shooting technique. At the end, the behavior of different parameters of interest are shown graphically.

2. Mathematical model

For an incompressible fluid the balance of mass and momentum are given by

$$\operatorname{div} \overline{\mathbf{V}} = 0, \tag{1}$$

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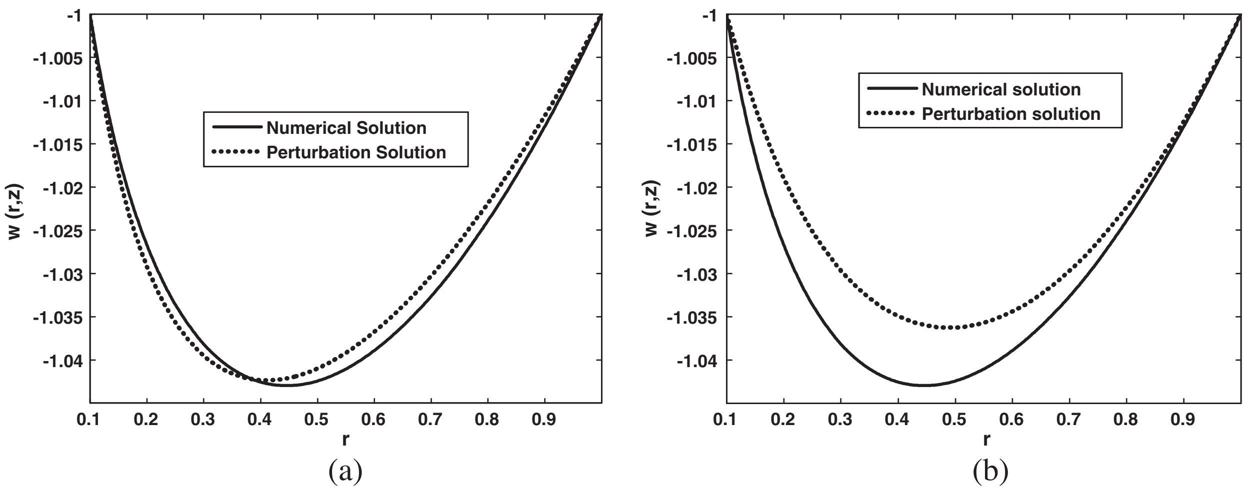

Fig. 1. Comparison of axial velocity for perturbation and numerical solutions when (a) $K = 0.1$, (b) $K = 0.5$, other parameters are $\phi = 0.4,\ \varepsilon = 0.1,\ \frac{dp}{dz} = 0.4,\ z = 0.5,\ M = 0.5$.

Table 1
Comparison of axial velocity for perturbation and numerical solutions when (a) $K = 0.1$, (b) $K = 0.5$, other parameters are $\phi = 0.4,\ \varepsilon = 0.1,\ \frac{dp}{dz} = 0.4,\ z = 0.5,\ M = 0.5$.

r	Numerical sol when $(K = 0.1)$	Perturb sol	Numerical sol when $(K = 0.5)$	Perturb sol
0.10	−1.000000	−1.000000	−1.000000	−1.000000
0.15	−1.013079	−1.014210	−1.016465	−1.010710
0.20	−1.020100	−1.021160	−1.029170	−1.018960
0.25	−1.026625	−1.030789	−1.037464	−1.025111
0.30	−1.030679	−1.031621	−1.042150	−1.031621
0.35	−1.045808	−1.038601	−1.045808	−1.035891
0.40	−1.036289	−1.040463	−1.047766	−1.038451
0.45	−1.037758	−1.041561	−1.048315	−1.039621
0.50	−1.038221	−1.041632	−1.048003	−1.037822
0.55	−1.037908	−1.040785	−1.046604	−1.038521
0.60	−1.036671	−1.037642	−1.044183	−1.037642
0.65	−1.034967	−1.036805	−1.041439	−1.029661
0.70	−1.032094	−1.033389	−1.037320	−1.028631
0.75	−1.029015	−1.029900	−1.033230	−1.025292
0.80	−1.024494	−1.024958	−1.027559	−1.022292
0.85	−1.020031	−1.020021	−1.022216	−1.018654
0.90	−1.013829	−1.014380	−1.015081	−1.014380
0.95	−1.007942	−1.007858	−1.007172	−1.006592
1.00	−1.000000	−1.000000	−1.000000	−1.000000

$$\rho \frac{d\overline{\mathbf{V}}}{dt} = -\nabla \overline{P} + \operatorname{div} \overline{\mathbf{S}}, \tag{2}$$

where ρ is the density, $\overline{\mathbf{V}}$ is the velocity vector, $\overline{P}$ is the pressure $\overline{\mathbf{S}}$ is the Cauchy stress tensor, and d/dt represents the material time derivative. The constitutive equation for Eyring–Powell fluid model is given by [16]

$$\overline{\mathbf{S}} = \mu \nabla \overline{\mathbf{V}} \frac{1}{\beta} \sinh^{-1}\left(\frac{1}{c} \nabla \overline{\mathbf{V}}\right) \tag{3}$$

and

$$\sinh^{-1}\left(\frac{1}{c}\nabla\overline{\mathbf{V}}\right) \approx \frac{1}{c}\nabla\overline{\mathbf{V}} - \frac{1}{6}\left(\frac{1}{c}\nabla\overline{\mathbf{V}}\right)^3, \quad \left|\frac{1}{c}\nabla\overline{\mathbf{V}}\right| \ll 1. \tag{3a}$$

where μ is the coefficient of shear viscosity, β and c are the material constants.

3. Mathematical formulation

We have considered peristaltic flow of an incompressible Eyring–Powell fluid in an endoscope. The flow is generated by sinusoidal wave trains propagating with constant speed c_1 along the wall of the upper tube. Heat and mass transfer phenomena have been consider giving temperature $\overline{T}_0,\ \overline{T}_1$ and concentration $\overline{C}_0$ and $\overline{C}_1$ to the inner and outer tube respectively.

The geometry of the wall surfaces is defined as

$$\overline{R}_1 = a_1, \tag{4a}$$

$$\overline{R}_2 = a_2 + b \sin \frac{2\pi}{\lambda}(\overline{Z} - c_1 \overline{T}), \tag{4b}$$

where a_1 is the radius of the inner tube, a_2 is the radius of the outer tube at inlet, b is the wave amplitude, λ is the wavelength, c_1 the wave speed and $\overline{T}$ the time. We are considering the cylindrical coordinate system $(\overline{R}, \overline{Z})$, in which $\overline{Z}$-axis lies along the center line of the tube and $\overline{R}$ is transverse to it.

The governing equations in the fixed frame for an incompressible flow are given as

$$\frac{\partial \overline{U}}{\partial \overline{R}} + \frac{\overline{U}}{\overline{R}} + \frac{\partial \overline{W}}{\partial \overline{Z}} = 0, \tag{5}$$

$$\rho\left(\frac{\partial}{\partial \overline{T}} + \overline{U}\frac{\partial}{\partial \overline{R}} + \overline{W}\frac{\partial}{\partial \overline{Z}}\right)\overline{U} = -\frac{\partial \overline{P}}{\partial \overline{R}} - \frac{1}{\overline{R}}\frac{\partial}{\partial \overline{R}}(\overline{R}\overline{\mathbf{S}}_{\overline{R}\overline{R}}) - \frac{\partial}{\partial \overline{Z}}(\overline{\mathbf{S}}_{\overline{R}\overline{Z}}) - \frac{\overline{\mathbf{S}}_{\overline{\theta}\overline{\theta}}}{\overline{R}}, \tag{6}$$

$$\rho\left(\frac{\partial}{\partial \overline{T}} + \overline{U}\frac{\partial}{\partial \overline{R}} + \overline{W}\frac{\partial}{\partial \overline{Z}}\right)\overline{W} = -\frac{\partial \overline{P}}{\partial \overline{Z}} - \frac{1}{\overline{R}}\frac{\partial}{\partial \overline{R}}(\overline{R}\overline{\mathbf{S}}_{\overline{R}\overline{Z}}) - \frac{\partial}{\partial \overline{Z}}(\overline{\mathbf{S}}_{\overline{Z}\overline{Z}}), \tag{7}$$

$$\rho c_p\left(\frac{\partial}{\partial \overline{T}} + \overline{U}\frac{\partial}{\partial \overline{R}} + \overline{W}\frac{\partial}{\partial \overline{Z}}\right)\overline{T} = \overline{\mathbf{S}}_{\overline{R}\overline{R}}\frac{\partial \overline{U}}{\partial \overline{R}} + \overline{\mathbf{S}}_{\overline{R}\overline{Z}}\frac{\partial \overline{W}}{\partial \overline{R}} + \overline{\mathbf{S}}_{\overline{Z}\overline{R}}\frac{\partial \overline{U}}{\partial \overline{Z}} + \overline{\mathbf{S}}_{\overline{Z}\overline{Z}}\frac{\partial \overline{W}}{\partial \overline{Z}} + k_2\left(\frac{\partial^2 \overline{T}}{\partial \overline{R}^2} + \frac{1}{\overline{R}}\frac{\partial \overline{T}}{\partial \overline{R}} + \frac{\partial^2 \overline{T}}{\partial \overline{Z}^2}\right), \tag{8}$$

$$\left(\frac{\partial}{\partial \overline{T}} + \overline{U}\frac{\partial}{\partial \overline{R}} + \overline{W}\frac{\partial}{\partial \overline{Z}}\right)\overline{C} = D\left(\frac{\partial^2 \overline{C}}{\partial \overline{R}^2} + \frac{1}{\overline{R}}\frac{\partial \overline{C}}{\partial \overline{R}} + \frac{\partial^2 \overline{C}}{\partial \overline{Z}^2}\right) + \frac{DK_T}{T_m}\left(\frac{\partial^2 \overline{T}}{\partial \overline{R}^2} + \frac{1}{\overline{R}}\frac{\partial \overline{T}}{\partial \overline{R}} + \frac{\partial^2 \overline{T}}{\partial \overline{Z}^2}\right), \tag{9}$$

where $\overline{P}$ is the pressure and $\overline{U},\ \overline{W}$ are the respective velocity components in the radial and axial directions in the fixed frame respectively, $\overline{C}$ is the concentration of fluid, T_m is temperature of the medium, D is the coefficients of mass diffusivity and K_T is the thermal diffusion ratio. In the fixed coordinates $(\overline{R},\ \overline{Z})$, the flow is