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Analysis of the turbulence–radiation interactions for large eddy simulations of turbulent flows

Maxime Roger, Carlos B. Da Silva, Pedro J. Coelho *

Mechanical Engineering Department, Instituto Superior Técnico/IDMEC, Technical University of Lisbon, Av. Rovisco Pais, 1049-001 Lisboa, Portugal

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ABSTRACT

In large eddy simulations (LES) of radiative heat transfer, the subgrid-scale (SGS) radiative emission and the SGS radiative absorption are two unclosed terms that arise after applying the filtering operation to the radiative transfer equation and which represent the effect of the SGS motions on the evolution of the resolved radiative heat flux. In the present work, *a-priori* tests based on direct numerical simulation of homogeneous isotropic turbulence have been carried out. It was found that the effect of the SGS radiative absorption may be neglected in LES, whereas the SGS radiative emission has to be modelled, particularly for engineering applications where the grids are generally coarse, and in flows with high turbulence intensities. Future works should be devoted to the development of SGS models for radiative transfer.

1. Introduction

The numerical simulation of turbulent flows is an important tool in the study of many engineering and natural flows such as the ones encountered in aerodynamics, turbomachinery, combustion, oceanography or astrophysics. Direct and large eddy simulations (DNS/LES) are the most advanced and accurate numerical techniques used for modelling these flows [1]. In DNS, all the relevant scales of motion are solved, from the integral scale down to the Kolmogorov micro-scale, whereas in LES, the large energetic scales of motion are explicitly computed and the effect of the subgrid-scales (SGS) on the resolved scales is modelled. Although being virtually unmatched by any other technique, DNS is not always feasible due to its high computational cost at high Reynolds numbers, while LES can be used to compute high Reynolds number flows at reasonable cost.

LES is fast becoming a realistic alternative to the classical simulations of reactive flows using unsteady Reynolds-averaged Navier–Stokes equations (URANS) [2]. LES of turbulent combustion are particularly challenging since the chemical reactions that govern the evolution of the species concentration take place at scales of motion which are substantially smaller than the scales resolved by the LES grid [3]. However, this difficult problem has been addressed by numerous works in recent years and today it is possible to perform LES involving turbulent combustion in devices as complex as a complete gas turbine combustor [4,5].

On the other hand, much less attention has been paid to radiative transfer when LES is employed to model the flow field [6], although it is well known that radiative transfer plays an important role in many combustion systems such as boilers, furnaces, internal combustion engines and rockets. The interactions between turbulence and radiation (TRI) have been studied both theoretically and experimentally [7] and various numerical works on TRI have been developed over the last years, using RANS [8–10], PDF methods [11–13], and DNS [14,15]. However, in the LES framework, very little is known about the interaction between the turbulence and radiation, *i.e.*, TRI for LES. Usually, in combustion applications of LES, these interactions are either discarded altogether, or included in the computations without considering any subgrid-scale model for radiation [16–19]. To the author's knowledge, there is no work available in the literature about TRI modelling in LES. The only exception is the recent study from Poitou et al. [20] in which two-dimensional *a-priori* tests were carried out to assess the influence of the temperature auto-correlation and the correlation between the temperature and the absorption coefficient on the radiation emission of a methane–air triple flame. In that study, simple models based on Taylor development were proposed to reconstruct these correlations in the radiative emission, and the radiative absorption was neglected.

The goal of the present paper is to study the influence of the unresolved scales, or subgrid-scales of motion, on the resolved or grid-scale radiative absorption and emission. In particular, the present investigation is directed on assessing the relevant terms in the filtered radiative source term. This is achieved through classical *a-priori* tests where the application of several filters to a DNS data bank is used to assess several of the exact quantities involved

* Corresponding author. Tel.: +351 21 841 81 94; fax: +351 21 847 55 45.
E-mail address: pedro.coelho@ist.utl.pt (P.J. Coelho).

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in LES. It is expected that the present results will guide the future modelling of TRI for LES.

The present work is a continuation of the previous work by Roger et al. [21] where the filtered radiative transfer equation (RTE) to use in LES was first studied. To simplify the analysis, combustion is not considered in the present work. We use as a starting point instantaneous fields of a passive scalar from a DNS of statistically stationary (forced) homogeneous isotropic turbulence [22]. The passive scalar field is then used to define the temperature and the chemical composition of the medium, which are decoupled from the radiative transfer calculations in the present work. Using these fields as input data, the thermal radiation is calculated with a ray-tracing/correlated k-distribution method. Finally, by applying a box or a cut-off filter to the data, classical *a-priori* tests are performed in order to evaluate the different terms arising from the filtered RTE.

This article is organized as follows. In the next section, the filtered RTE to solve in LES of radiative heat transfer is described. The new terms associated with the SGS radiative transfer and that require modelling are identified. In Section 3, the numerical tools for turbulence and for radiation calculations are detailed, as well as the assumptions and the models involved in these calculations. In Section 4, the results are presented and discussed. Finally, the article ends with an overview of the main results and conclusions.

2. LES of thermal radiation: the filtered radiative transfer equation

In LES, any given flow variable $Q(\mathbf{r},t)$ is decomposed into a resolved or grid-scales (GS) part $\overline{Q}(\mathbf{r},t)$, and a residual or subgrid-scales (SGS) part $Q''(\mathbf{r},t)$, through a spatial low-pass filtering operation defined as [1]

$$\overline{Q}(\mathbf{r},t) = \int Q(\mathbf{x},t)\,G(\mathbf{r}-\mathbf{x})\,d\mathbf{x} \tag{1}$$

where $G(\mathbf{r})$ represents the filter function. Therefore, any given flow variable can be decomposed as

$$Q(\mathbf{r},t) = \overline{Q}(\mathbf{r},t) + Q''(\mathbf{r},t) \tag{2}$$

The Navier–Stokes equations in the case of a viscous incompressible flow may be written as

$$\frac{\partial u_i}{\partial t} + \frac{\partial(u_i u_j)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \tag{3}$$

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{4}$$

Applying the filter operation, one obtains the filtered Navier–Stokes equations

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial(\bar{u}_i \bar{u}_j)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right)\right] - \frac{\partial \tau_{ij}}{\partial x_j} \tag{5}$$

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \tag{6}$$

where u_i and p are the velocity and pressure fields, respectively, μ is the dynamic viscosity, ρ the density and $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i\bar{u}_j$ is the subgrid-scale stresses tensor, which describes the effect of the subgrid-scales on the resolved scales of motion and that has to be modelled. For instance, a subgrid-scale model frequently used for τ_{ij} is the well known Smagorinsky model [23]. In this model, the subgrid-scale stresses tensor is closed by assuming local equilibrium between the production and the viscous dissipation of subgrid-scale kinetic energy, and by using an eddy-viscosity assumption,

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2\nu_t(\mathbf{r},t)\overline{S}_{ij} \tag{7}$$

where $\overline{S}_{ij} = \frac{1}{2}(\partial\bar{u}_i/\partial x_j + \partial\bar{u}_j/\partial x_i)$ is the resolved rate-of-strain tensor, and $\nu_t(\mathbf{r},\ t)$ is the turbulent eddy-viscosity which is modelled as $\nu_t(\mathbf{r},t) = (C_S\Delta)^2|\overline{S}|$. C_S is the Smagorinsky model constant and $|\overline{S}| = (2\overline{S}_{ij}\overline{S}_{ij})^{1/2}$ is the norm of the resolved rate-of-strain tensor. In isotropic turbulence the constant C_S is related to the Kolmogorov constant C_K through

$$C_S = \frac{1}{\pi}\left(\frac{2}{3C_k}\right)^{3/4} \tag{8}$$

Using $C_K = 1.4$, we get $C_S = 0.18$.

Similarly, in LES of non-isothermal flows, in addition to the filtered Navier–Stokes equations, a filtered temperature transport equation has to be solved,

$$\rho c_p\left(\frac{\partial \overline{T}}{\partial t} + \frac{\partial(\bar{u}_j\overline{T})}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left[\lambda\frac{\partial \overline{T}}{\partial x_j}\right] - \rho c_p\frac{\partial F_j}{\partial x_j} - \overline{\nabla\cdot\mathbf{q}} \tag{9}$$

where $\overline{T}$ is the resolved temperature field, λ is the thermal conductivity, c_p is the specific heat capacity and $F_j = \overline{Tu_j} - \overline{T}\bar{u}_j$ are the SGS heat fluxes requiring modelling. When present, the effects of thermal radiation arise as an additional radiative source term $\nabla\cdot\mathbf{q}$ given by [24]

$$\nabla\cdot\mathbf{q} = \int_0^{+\infty}\kappa_\nu(4\pi I_{b\nu} - G_\nu)\,d\nu \tag{10}$$

where $\mathbf{q}$ is the radiative heat flux vector and $G_\nu = \int_{4\pi} I_\nu d\Omega$ is the spectral incident radiation, which is obtained by integrating $I_\nu(\mathbf{r},\mathbf{s})$ – the spectral radiation intensity at point $\mathbf{r}$ and direction $\mathbf{s}$ – over all directions. $I_{b\nu}(\mathbf{r})$ is the spectral blackbody radiation intensity and κ_ν the spectral absorption coefficient.

The spectral radiation intensity $I_\nu(\mathbf{r},\mathbf{s})$ is governed by the radiative transfer equation (RTE) which for an emitting–absorbing and non-scattering medium may be written as

$$\frac{dI_\nu(\mathbf{r},\mathbf{s})}{ds} = -\kappa_\nu(\mathbf{r})I_\nu(\mathbf{r},\mathbf{s}) + \kappa_\nu(\mathbf{r})I_{b\nu}(\mathbf{r}) \tag{11}$$

The filtered radiative transfer equation is obtained by applying a spatial filtering operation to Eq. (11)

$$\begin{aligned}\frac{d\bar{I}_\nu}{ds} &= -\overline{\kappa_\nu I_\nu} + \overline{\kappa_\nu I_{b\nu}} \\ &= -\bar{\kappa}_\nu\bar{I}_\nu - (\overline{\kappa_\nu I_\nu} - \bar{\kappa}_\nu\bar{I}_\nu) + \bar{\kappa}_\nu\bar{I}_{b\nu} + (\overline{\kappa_\nu I_{b\nu}} - \bar{\kappa}_\nu\bar{I}_{b\nu})\end{aligned} \tag{12}$$

where the position $\mathbf{r}$ and the spatial direction $\mathbf{s}$ were omitted for simplicity. The terms $(\overline{\kappa_\nu I_\nu} - \bar{\kappa}_\nu\bar{I}_\nu)$ and $(\overline{\kappa_\nu I_{b\nu}} - \bar{\kappa}_\nu\bar{I}_{b\nu})$ are the spectral radiative SGS heat fluxes and have to be modelled in order to close Eq. (12).

In the following analysis, we have focused on radiative quantities integrated over the spectrum, like the Planck mean absorption coefficient κ_P and the incident mean absorption coefficient κ_G, which are defined as follows

$$\kappa_P = \frac{\int_0^{+\infty}\kappa_\nu I_{b\nu}d\nu}{\int_0^{+\infty}I_{b\nu}d\nu} \tag{13}$$

$$\kappa_G = \frac{\int_0^{+\infty}\kappa_\nu G_\nu d\nu}{\int_0^{+\infty}G_\nu d\nu} \tag{14}$$

It was assumed that the radiation is isotropic to be consistent with the assumption of isotropic and homogeneous turbulence, which allows us to write

$$\kappa_G = \frac{\int_0^{+\infty}\kappa_\nu G_\nu d\nu}{\int_0^{+\infty}G_\nu d\nu} \approx \frac{\int_0^{+\infty}\kappa_\nu I_\nu d\nu}{\int_0^{+\infty}I_\nu d\nu} \tag{15}$$

Notice that, in practical problems, the assumption of isotropic radiation is approximately valid for perfectly stirred reactors. Integration of Eq. (12) over the spectrum yields