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A comparison of models and methods for simulating the microwave heating of moist foodstuffs

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ABSTRACT

We study the problem of heating a one-dimensional approximation to a slab-sided moist foodstuff in a microwave oven, allowing for a phase change and drying. We initially investigate the accuracy of the Lambert law of exponential decay of the applied electric field into the foodstuff and derive an approximation for the field comprising the exponential decay term and an oscillatory component. We then show that the temperature of the foodstuff is given, to a good approximation, by only considering the heating effects of the exponentially decaying field. We then study the effects of drying. This process changes the dielectric properties of the material, which leads to changes in the field. However, these lead to smaller changes in the moisture content. A fast and accurate numerical method is derived which relies on smoothing the phase transition.

1. Introduction

Microwave ovens are frequently used in domestic situations for the heating of chilled foodstuffs which are generally approximately 80% by weight water. Rapid, internal heating are some of the key benefits over conventional ovens. The food industry utilises these attributes in a number of ways most in particular with the introduction of microwave ready meals and convenience food. It is important that the food properly heated to ensure that is micro-biologically safe for consumption. One of the principle modes of heating in microwave cooking is through dipole orientation. In the case of foodstuffs there is a large concentration of polar water molecules, when exposed to an electromagnetic field these molecules attempt to align themselves with the field. Domestic microwave ovens typically use an EM frequency of 2.45 GHz and so the water attempts to line up with a rapidly changing field. Internal reflections within the oven cavity can lead to standing wave patterns forming in the EM field, and these troughs in the field can lead to so called "cold spots" in the food. It is in these regions of low temperature that harmful organisms can propagate and survive.

One of the main difficulties in modelling the processes involved in microwave cooking is that of determining the electromagnetic field both inside and outside the foodstuff. Maxwell's equations can be difficult to solve and the continuously changing field patterns require a new field solution to be calculated frequently. Full three dimensional electromagnetic calculations of the solution to Maxwell's equations must then be coupled to a suitable model for heat and moisture transport within a foodstuff, taking into account the phase changes within the foodstuff and also the change in the various dielectric constants. Such calculations can take many hours [1] and it is consequently difficult to use these approaches to consider the effects of parameter variations in the design of microwave heating devices. An effective approach to speeding up these computations is to derive simplified, yet accurate, approximations to both the electromagnetic field and to the resulting heating patterns in the food. This can result in very significant speed ups of the calculations, although this is at the expense of a degree of accuracy in the calculations. The purpose of this paper is to study certain of these approximate models, to derive estimates on their accuracy and suitability for modelling and to determine efficient numerical methods to find useful approximations to their solution. A common approximation to the field within a heated sample, known as Lambert's law, is to assume an exponential decay of the field intensity with depth from the surface of the foodstuff. Lambert's law is derived from Maxwell's equations in one dimension [2], and the model is valid for semi infinite domains. The Lambert law model can be used to model the moisture changes within a heated sample [3,4]. The work carried out by Ni et al. [3] results in an extensive moisture transport model for microwave heating in one dimension. Significantly the dielectric properties of a sample are found to be highly dependent on the moisture content. When the electric field is calculated using Lambert's law, the dielectric properties must be recalculated to reflect the changing moisture content of the sample. It is found that the field was able to penetrate further into a dry sample than a wet one, because as the moisture content decreases, the materials ability to absorb microwave energy also decreases. This leads to a change in

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the electric field and the power absorbed by the sample. This will have an effect on the temperature and moisture content of the load. However, the Lambert law approximation to the field does not take into account the internal reflections which can occur in shorter sample lengths where the internal reflected waves interfere with the incident waves resulting in a standing wave pattern. The resulting fields take the form of an oscillating electric field intensity centred around an exponential decay [5]. A comparison of Lambert's law and the exact solution to Maxwell's equations is given in the paper by Ayappa et al. [6]. Basak [7] extends the work of Ayappa to conduct an analysis on a multilayered material which undergoes a phase change from frozen to liquid. The numerical studies find spatial resonance patterns in the multilayered slabs.

In this paper we study a simple model of the microwave heating of a chilled and moist foodstuff, with the main example being mashed potato which comprises a mixture of starch and water. When heating such a foodstuff the moisture content remains nearly constant until a temperature of $T_b = 100$ °C is reached at which point the starch starts to dry out, leading to a change in the moisture content and a consequent change in the dielectric properties of the foodstuff (which also depend weakly upon temperature) [8,9]. The purpose of the analysis presented in this paper is twofold. Firstly we compare and contrast the foodstuff temperature profiles that result from using Maxwell's equations for the field, from those given by the Lambert law. Secondly we consider the differences between the temperature and moisture profiles that arise when using constant dielectric properties with those that arise with dielectric properties which depend upon temperature and moisture content. We consider both an analytic approach and also a numerical approach related to a smoothed form of the enthalpy method. For the purposes of this investigation, we investigate the oscillations in the field and temperature inside a one dimensional sample of food. This is a reasonable first approximation to a slab-sided foodstuff.

The main conclusions of this paper are that, for the typical wavelength of microwaves used in cooking, the difference in the temperature profile calculated from using the Lambert law approximation for the field from that calculated by solving Maxwell's equations, are small provided that the foodstuff is more than 2 cm in extent. The difference is manifest as an oscillation about a decaying solution and the relative amplitude of this temperature oscillation is significantly smaller than the oscillations of the field strength around the exponential decay profile. We also conclude that there is not a particularly significant difference in the temperature and moisture content profiles that arise when using constant values for the dielectric parameters from those given by variable parameters. A further conclusion is that the smoothed numerical method we employ gives a fast and accurate way of calculating the temperature and moisture profiles provided that the smoothing parameter is chosen carefully.

The layout of this paper is as follows. In Section 2 we outline the basic theory for the field equations interior to the foodstuff and derive both the Lambert law approximation and the oscillatory correction to this. In Section 3 we determine the resulting temperature distribution of a two-phase moist foodstuff. In Section 4 we determine the effects of dielectric variation under changes in moisture and temperature and make a numerical calculation of the resulting temperature by using a version of the smoothed enthalpy method. Finally in Section 5 we draw some further conclusions from this work.

2. The field distribution

2.1. Lambert's law

We will consider a one-dimensional foodstuff with microwave radiation incident from the left and with the boundary of the foodstuff at the position $x = 0$. (This is a not unreasonable approximation to the geometry of a slab-sided food in a microwave oven.) Initially we assume that the foodstuff occupies the whole region $0 \leqslant x$ with x the distance into the food. The electric field intensity E of the electromagnetic field with frequency, ω, obeys Maxwell's equations, which in a one-dimensional medium reduce to the Helmholtz equation [6].

$$E_{xx} + \lambda^2 E = 0, \tag{1}$$

where

$$\lambda^2 = \omega^2 \mu(x) \varepsilon_0 \kappa^*(x) \tag{2}$$

and $\mu(x)$ is the magnetic permeability of the propagating medium, ε_0 is the permittivity of free space and $\kappa^*(x) = \kappa'(x) + i\kappa''(x)$ is the complex dielectric. Setting

$$\lambda = \alpha + i\beta,$$

yields the attenuation coefficient

$$\beta = \omega\sqrt{\mu\varepsilon_0}\sqrt{\frac{\kappa'\left(\sqrt{1+\tan^2(\delta)}-1\right)}{2}}, \quad \tan(\delta) = \frac{\kappa''}{\kappa'}. \tag{3}$$

We now compare two solutions to the above Helmholtz equation making the initial assumption that the dielectric properties of the foodstuff remain constant in space and time throughout the heating process. The general solution of (1) for constant λ is

$$E = A_0 e^{i\lambda x} + B_0 e^{-i\lambda x}, \tag{4}$$

$$E = A_0 e^{i\alpha x} e^{-\beta x} + B_0 e^{-i\alpha x} e^{\beta x}. \tag{5}$$

When considering a semi-infinite domain $0 \leqslant x < \infty$ we must impose the condition $B_0 = 0$ to prevent $|E| \to \infty$ as $x \to \infty$. The power P absorbed by a sample per unit volume is given by Metaxas [2]

$$P = \frac{1}{2}\omega\varepsilon_0\kappa''|E|^2, \tag{6}$$

so that in this case

$$P = Q_0 e^{-2\beta x}, \tag{7}$$

where Q_0 is the power density at the surface of the material exposed to the electro-magnetic field. This description of the exponentially decaying power is precisely the Lambert law. For a typical moist starchy foodstuff

$$\alpha \approx 450 \text{ m}^{-1} \quad \text{and} \quad \beta \approx 60 \text{ m}^{-1}.$$

2.2. The power absorbed by a finite section of foodstuff

We will now summarise some of the work of Ayappa [6] who derived expressions for the power absorbed by a *finite section* of foodstuff occupying the region $0 \leqslant x \leqslant L$ and will compare this with the predictions of Lambert's law for varying lengths of domain. This formulation will take into account internal reflections within the foodstuff. We examine a section of foodstuff as in the diagram below, Fig. 1.

We denote the region to the left of the foodstuff with the subscript 1, the region within the foodstuff by 2 and the region to the right of the foodstuff as 3, and use subscripts on all coefficients to represent the appropriate region. In each of these regions the Helmholtz equation for the electric field intensity, (1), holds and so the electric field is

$$E_n = A_n e^{i\lambda_n x} + B_n e^{-i\lambda_n x} \tag{8}$$

for regions $1 \leqslant n \leqslant 3$. The first term in the expression describes the wave traveling from left to right, the second describes the wave traveling from right to left. We impose the following continuity