International Journal of Heat and Mass Transfer 54 (2011) 854–862

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Radiation effects on MHD flow of Maxwell fluid in a channel with porous medium

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ARTICLE INFO

Article history:
Received 23 April 2010
Received in revised form 22 September 2010
Accepted 27 September 2010
Available online 8 November 2010

Keywords:
Maxwell fluid
Heat transfer
Porous medium
Porous channel

ABSTRACT

This paper describes the heat transfer analysis with thermal radiation on the two-dimensional magnetohydrodynamic (MHD) flow in a channel with porous walls. The upper-convected Maxwell (UCM) fluid fills the porous space between the channel walls. The corresponding boundary layer equations are transformed into ordinary differential equations by means of similarity transformations. The resulting problems are solved by employing homotopy analysis method (HAM). Convergence of the derived series solutions is ensured. The effects of embedded parameters on the dimensionless velocity components and temperature are examined through plots. The variation of local Nusselt number is also analyzed.

1. Introduction

The flows in porous channels/tubes are of special interest in the several applications in biomedical and mechanical engineering. Such flows appear in the blood dialysis in artificial kidney, flow in the capillaries, flow in blood oxygenators, the design of filters and design of porous pipe. Many fluids of industrial importance are non-Newtonian. An extension of flow analysis from viscous to the non-Newtonian fluids is not so straightforward. In fact, the difficulties occur by the diversity of non-Newtonian fluids in their constitutive relationship and simultaneous effects of viscosity and elasticity. These viscoelastic effects add complexities in the resulting differential equations. Some interesting contributions on the topic can be found in the studies [1–15] and several references therein.

The non-Newtonian fluids are mainly classified into three types namely differential, rate and integral. The simplest subclass of the rate type fluids is the Maxwell model [16]. This fluid model can very well describe the relaxation time effects. It is worthmentioning to point out that Maxwell did not developed his model for polymeric liquids, but instead for air, the methodology used by him has been extended by Rajagopal and Srinivasa [17] to produce a plethora of rate type models [18]. Choi et al. [19] discussed the steady hydrodynamic boundary layer flow of an incompressible Maxwell fluid in a porous channel. Abbas et al. [20] reported magnetohydrodynamic effects on the flow analysis presented in a study [19]. Hayat et al. [21,22] extended this analysis for the second grade and Jeffery fluids. Then Hayat and Abbas [23] discussed boundary layer flow of an incompressible Maxwell fluid in porous channel with chemical reaction.

The aim of present attempt is to venture further in this regime. For that we have an interest to examine the steady boundary layer flow of an upper-convected Maxwell fluid in a porous channel with heat transfer analysis when radiation effects are present. An incompressible fluid saturates the porous medium. The resulting nonlinear problem is treated for a series solution by homotopy analysis method (HAM) [24–45]. Convergence of the HAM solution is established and the variations of emerging parameters are highlighted on the velocity and temperature.

2. Definition of the problem

Let us examine the heat transfer characteristics on MHD two-dimensional flow of an incompressible upper-convected Maxwell fluid in a channel with porous walls at $y = \pm H/2$ (see, Fig. 1). Porous medium fills the space between the walls of the channel. Flow is induced by suction/blowing. A constant magnetic field $\mathbf{B}_0$ is applied in the y-direction and there is no external electric field. The induced magnetic field is neglected under the assumption of small magnetic Reynolds number. Furthermore, symmetric nature of the flow is taken into account and pressure gradient is neglected.

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0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2010.09.069

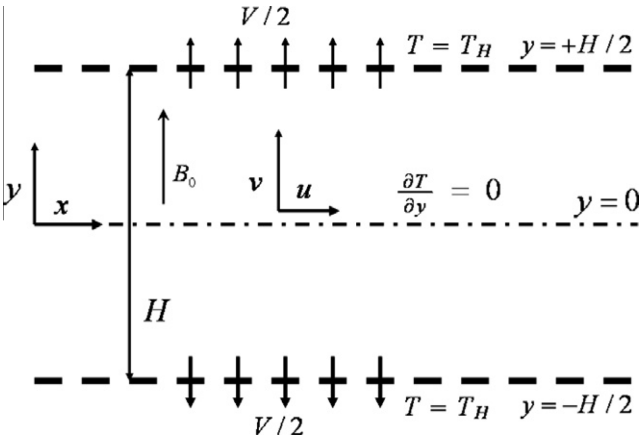

Fig. 1. Physical model.

Both the walls have same temperature T_H. The temperature at the centerline ($y = 0$) is $\frac{\partial T}{\partial y} = 0$. The boundary layer equations for the flow under consideration are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda\left[u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x \partial y}\right] = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}\left(u + \lambda v\frac{\partial u}{\partial y}\right) - \frac{\phi\nu}{k}u, \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_0\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

in which u and v are the velocity components in the x and y-directions respectively, ρ is the fluid density, ν is the kinematic viscosity, σ is the electrical conductivity, ϕ is the porosity, k is the permeability of the porous medium, λ is the relaxation time, c_p is the specific heat at constant temperature, k_0 is the thermal conductivity of the fluid, T is the temperature and q_r is the radiative heat flux. Further, it is pointed out that Eq. (2) is a correct version of the equation of motion in the previous studies [20,46–52].

Using the Rosseland approximation for radiation in an optically thick layer [53] one obtains

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{4}$$

where σ^* is the Stefan–Boltzmann constant and k^* is the mean absorption coefficient. By Taylors' series about T_H, T^4 can be written as

$$T^4 \cong 4T_H^3 T - 3T_H^4. \tag{5}$$

With the help of Eqs. (4) and (5), Eq. (3) becomes

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \left(k_0 + \frac{16\sigma^* T_H^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2. \tag{6}$$

The relevant boundary conditions are

$$\frac{\partial u}{\partial y} = 0, \quad v = 0, \quad \frac{\partial T}{\partial y} = 0 \quad \text{at} \quad y = 0, \tag{7}$$

$$u = 0, \quad v = \frac{V}{2}, \quad T = T_H \quad \text{at} \quad y = \frac{H}{2} \tag{8}$$

in which $V > 0$ corresponds to suction and $V < 0$ indicates blowing.

Invoking the following non-dimensional parameters [19]

$$x^* = \frac{x}{H}, \quad y^* = \frac{y}{H}, \quad u = -Vx^* f'(y^*), \quad v = Vf(y^*),$$
$$\theta(y^*) = \frac{T}{T_H}, \tag{9}$$

the similarity equations resulting from Eqs. (2) and (6) are

$$f''' - M(Ref' + Deff'') - Kf' + Re(f'^2 - ff'') + De(2ff'f'' - f^2 f''') = 0, \tag{10}$$

$$\left(1 + \frac{4}{3}Rd\right)\theta'' - PrRef\theta' + PrEcf''^2 = 0. \tag{11}$$

Eqs. (7) and (8) now give

$$f = 0, \quad f'' = 0, \quad \theta' = 0 \quad \text{at} \quad y = 0, \tag{12}$$

$$f = \frac{1}{2}, \quad f' = 0, \quad \theta = 1 \quad \text{at} \quad y = \frac{1}{2} \tag{13}$$

in which

$$M = \frac{\sigma B_0^2 H}{\rho V}, \quad K = \frac{H^2\phi}{k}, \quad Re = \frac{HV}{\nu}, \quad De = \frac{\lambda V^2}{\nu},$$

$$Pr = \frac{c_p\mu}{k_0}, \quad Ec = \frac{V^2 x^2}{c_p(T_H)}, \quad Rd = \frac{4\sigma^* T_H^3}{k_0 k^*} \tag{14}$$

are respectively called the Hartman number M, the porosity parameter K, the Reynolds number Re, the Deborah number De, the Prandtl number Pr, the Eckert number Ec and radiation parameter Rd. It is worth mentioning to note that $Re > 0$ corresponds to suction case and $Re < 0$ for blowing. For Newtonian fluid $De = 0$.

The local Nusselt number Nu_x is defined as

$$Nu_x = \frac{xq_w}{k_0(T_H)} = \frac{-x\left[\left(k_0 + \frac{16\sigma^* T_H^3}{3k^*}\right)\frac{\partial T}{\partial y}\right]_{y=1/2}}{k_0(T_H)} = -Re^{1/2}\left(1 + \frac{4}{3}Rd\right)\theta'(1/2). \tag{15}$$

In next section, series solutions for problem given by Eqs. (10)–(13) will be constructed by the homotopy analysis method.

3. Homotopy analysis solutions

For HAM solutions, the set of base functions for $f(y)$ and $\theta(y)$ can be expressed, respectively by

$$\{y^{2n+1}, n \geqslant 0\}, \tag{16}$$

$$\{y^{2n}, n \geqslant 0\}, \tag{17}$$

in the form

$$f(y) = \sum_{n=0}^{\infty} a_n y^{2n+1}, \tag{18}$$

$$\theta(y) = \sum_{n=0}^{\infty} b_n y^{2n}, \tag{19}$$

where a_n and b_n are the coefficients to be determined. The initial guesses $f_0(y)$ and $\theta_0(y)$ are chosen as

$$f_0(y) = y\left(\frac{3}{2} - 2y^2\right), \tag{20}$$

$$\theta_0(y) = 1. \tag{21}$$

The auxiliary linear operators $\mathcal{L}_f$ and $\mathcal{L}_\theta$ with their properties are given below