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Mixing-dynamics of a passive scalar in a three-dimensional microchannel

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ABSTRACT

The mixing of a diffusive passive-scalar driven by electro-osmotic fluid motion in a micro-channel is studied numerically. Secondary time-dependent periodic or random electric fields, orthogonal to the main stream, are applied to generate cross-sectional mixing. This investigation focuses on the mixing dynamics and its dependence on the frequency (period) of the driving mechanism. For periodic flows, the probability density function (PDF) of the scaled concentration settles into a self-similar curve showing spatially repeating patterns. In contrast, for random flows there is a lack of self-similarity in the PDF for the time interval considered. An exponential decay of the variance of the concentration, and associated moments, is found to exist for both periodic and random velocity fields. The numerical results also indicate that measures of chaoticity (in a deterministic chaotic system) decay exponentially in the frequency – at large frequencies – in agreement with the theory.

1. Introduction

The present work continues a series of studies on the mixing dynamics of an electrolyte solution flowing in a micro-channel [1–3]. These studies were motivated by the need of efficient and rapid mixing for biological analyses and rapid medical diagnosis [4,5]. A wide variety of passive and active methods have been used to generate mixing in micro-devices [6,7,5,8]. An active micro-mixer was proposed in [1], in which the mixing was obtained with time-dependent transverse electric fields to generate either periodic or random velocity fields. The numerical results showed a dependence of the degree of chaos in the system on the period of the advecting flow, as well as the strength of randomization. It was found that by decreasing the frequency and/or increasing the strength of the stochasticity (which would obviously generate better mixing), undesired Taylor dispersion effects were minimized. Chaotic flows occur when material lines and surfaces are stretched exponentially. The dependence of the degree of mixing on the frequency of the advecting flow described above (and also exhibited in other similar flows) is currently understood in a fairly qualitative sense.

The decay properties of the variance in concentration, without sources or fluxes at the boundaries, have been commonly used to quantify the mixing of a diffusive passive scalar quantity that is advected by a fluid. For example, studies of two-dimensional fully chaotic flows, have shown to exhibit an exponential decay of scalar variance in the long time limit [9–11]. The exponential divergence of nearby trajectories is characterized by the local finite-time Lyapunov exponent σ_L, defined as the logarithm of stretching divided by time for this problem. The distribution of σ_L, usually expressed by the probability density function, has also being used to explain the rate of decay in scalar variance [11,12]. It has been argued that if the domain scale is significantly larger than the flow scale, the description based on Lyapunov exponents alone can be inadequate for the prediction of decay rates during the final stage of mixing [13,14].

Accurate experimental measurements of [15,16] in chaotic two-dimensional time-periodic flows revealed an exponential decay in tracer variance, with evidence for persistent spatial patterns in the concentration field. These repeating patterns are also known as *strange eigenmodes*; at the late stages of mixing, the scalar represents a periodic eigenfunction of the linear advection–diffusion equation. The appearance of self-similar asymptotic probability density function (PDF) of the scalar field normalized by its variance, suggests that strange or statistical eigenmodes appear in flows with aperiodic time-dependence and has been examined in detail in [10,17–19]. Camassa et al. [20] studied the PDF for a

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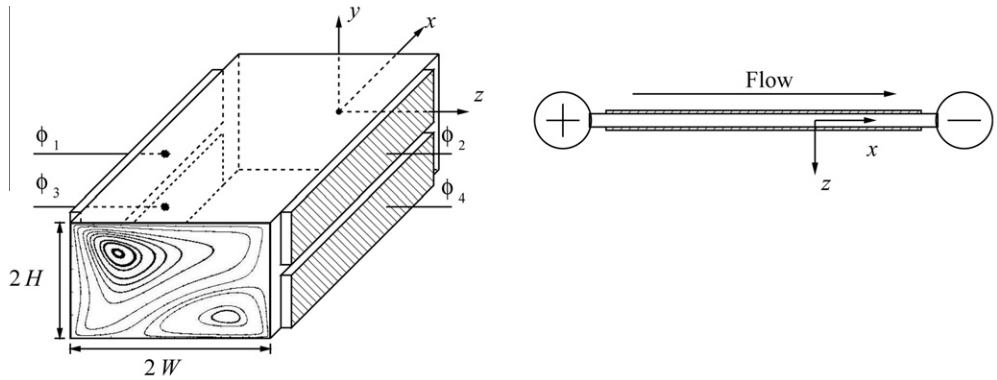


Fig. 1. Schematic of the flow apparatus to generate the velocity fields. The inset shows the streamlines of the transverse velocity field $\boldsymbol{u}_T$ for $(\phi_1, \phi_2, \phi_3, \phi_4) = (0,0,1,0)$. On the right, the primary flow is driven by the velocity generated by the electric potential ζ, at $y = \pm\Gamma$.

decaying passive scalar advected by deterministic velocities. They found a lack of self-similarity in the PDFs with time-periodic flows and self-similar PDFs in steady flows. Liu and Haller [18] have shown among other things, that for this advection–diffusion problem, a finite-dimensional inertial manifold exist, in which the decay of the tracer at the late stages of mixing is governed by the structure of the slowest-decaying mode in the manifold.

Our study addresses two important issues that to the best of our knowledge have not been documented before. The first is the appearance of strange eigenmodes in three-dimensional flows. The second focuses on connecting the theory regarding measures of chaoticity and its exponential decay (in a deterministic chaotic system) with results from numerical simulations; such connection is conspicuously lacking [21–24].

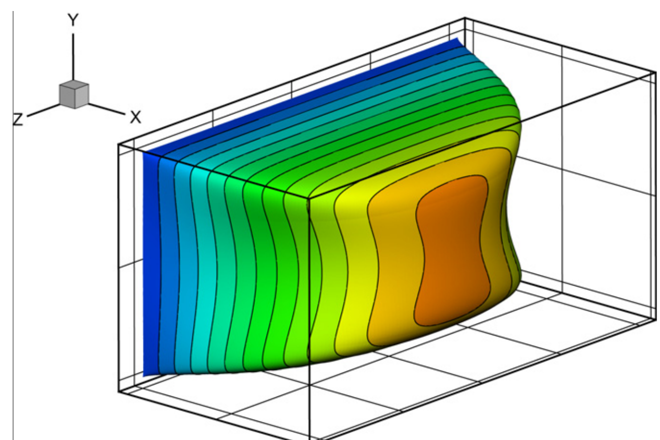
As in [3], we assume that the zeta potential at the lateral electrode walls is zero. Due to the capacitive charging at the electrode, there is a field-induced electro-osmosis – termed as AC electro-osmosis – and the zeta potential is non-zero [25,26]. Since the velocity profiles for AC electro-osmosis in perfectly polarizable electrodes is zero at high and low frequencies, and maximum at some intermediate characteristic frequency [27], in order to quantify the sole effect of the period modulation of the electric field in the mixing process, the effects of the field-induced electro-osmosis are not considered in our model. As it will be shown in the following sections, this simplified model reveals novel features not present in previous mixing mathematical formulations. Specifically, it is demonstrated that spatially repeating patterns develop for periodic flows, and that an exponential decay of variance in concentration exists. Additionally, quantitative measures of chaoticity for both periodic and random flows are provided.

The article is organized as follows: the governing equations and numerical integration scheme are briefly presented in Section 2. The results and discussion from the numerical simulations are presented in Section 3. A summary is presented in Section 4, which concludes the paper.

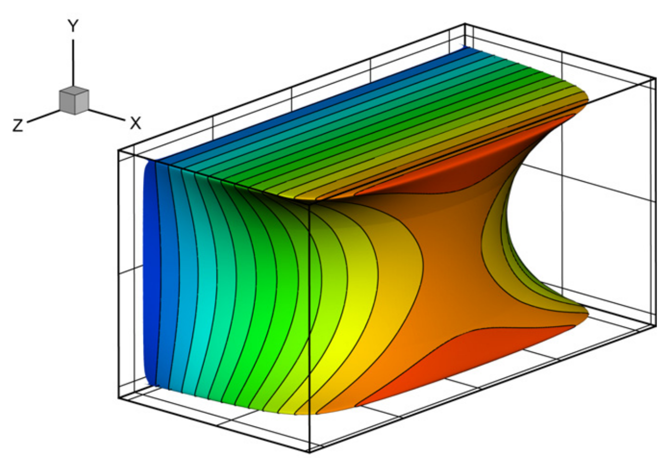
2. Governing equations and numerical method

We consider the electro-osmotic flow (EOF) inside a long rectangular micro-channel of height $2H$, and width $2W$ as shown in Fig. 1. This is the EOF micromixer studied by Pacheco and co-workers [1–3], where the typical dimensions for the device are $10\,\mathrm{cm} \times 100\,\mu\mathrm{m} \times 200\,\mu\mathrm{m}$. The primary steady flow along the channel is driven by a steady electric field, generated by zeta-potentials on the top and bottom surfaces. An unsteady transverse motion of the fluid is driven by secondary electric fields generated with four micro-electrodes placed on the lateral walls.

In Cartesian coordinates, (x,y,z) with their corresponding unit vectors $(\boldsymbol{i},\boldsymbol{j},\boldsymbol{k})$, the non-dimensional velocity vector and pressure are denoted by $\boldsymbol{u}$ and p, respectively. The non-dimensional electric potentials due to an external electric field is denoted by ϕ and the electric potential due to the electric charge at the walls by ψ. This



(a) $K^{-1} = 10^{-1}$.



(b) $K^{-1} = 10^{-2}$.

Fig. 2. Three-dimensional view of $\vec{u}_L = u(y,z,t)\vec{i}$ along the channel. (a) The maximum velocity occurs near the center of the channel and the slip-velocity model for this EOF is not valid; (b) the highest velocity occurs near the central portion of the upper and lower plates and the slip-velocity model may be valid.