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Flow and heat transfer of a third grade fluid past an exponentially stretching sheet with partial slip boundary condition

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ABSTRACT

Non-Newtonian boundary layer flow and heat transfer over an exponentially stretching sheet with partial slip boundary condition has been studied in this paper. The flow is subject to a uniform transverse magnetic field. The heat transfer analysis has been carried out for two heating processes, namely (i) with prescribed surface temperature (PST), and (ii) prescribed heat flux (PHF). Suitable similarity transformations are used to reduce the resulting highly nonlinear partial differential equations into ordinary differential equations. An effective second order numerical scheme has been adopted to solve the obtained differential equations. The important finding in this communication is the combined effects of the partial slip and the third grade fluid parameters on the velocity, skin-friction coefficient and the temperature boundary layer. It is found that the third grade fluid parameter β increases the momentum boundary layer thickness and decreases the thermal boundary layer thickness.

1. Introduction

The study of laminar boundary layer flow over a stretching sheet has received considerable attention in the past due to its applications in the industries, for example, materials manufactured by extrusion process, the boundary layer along a liquid film in condensation process and the heat treated materials traveling between a feed roll and the wind-up roll or on conveyor belt poses the features of a moving continuous surface. The flow and heat transfer phenomena over stretching surface have promising applications in a number of technological processes including production of polymer films or thin sheets. The no-slip boundary condition (the assumption that a liquid adheres to a solid boundary) is one of the central tenets of the Navier–Stokes theory. However, there are situations wherein this condition does not hold. Partial velocity slip may occur on the stretching boundary when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. Navier [1] proposed a slip boundary condition wherein the slip depends linearly on the shear stress. The inadequacy of the no-slip condition is evident for most non-Newtonian fluids. One can refer to the works of Andersson [2], Sahoo [3,4], Sahoo and Do [5], Ariel [6], Sajid et al. [7] and all the references therein regarding the flow and heat transfer of Newtonian and different non-Newtonian fluids past stretching sheets with slip and no-slip boundary conditions. Further the effects of slip and non-Newtonian flow parameters on the boundary layer flows can be seen in [8–10].

Elbashbeshy [11] has added a new dimension in his investigation by considering the flow and heat transfer of a Newtonian fluid over an exponentially stretching continuous surface. He considered an exponential similarity variable and exponential stretching velocity distribution on the coordinate considered in the direction of stretching. Partha et al. [12] have discussed the effects of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface in a quiescent viscous fluid. Sajid and Hayat [13] have investigated the influence of thermal radiation on the boundary layer flow and heat transfer of an incompressible viscous fluid due to an exponentially stretching sheet. Recently, Pal [14] has carried out an analysis to describe mixed convection heat transfer in the boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution in the presence of magnetic field, viscous dissipation and internal heat generation/absorption. Further one can refer to the works of Al-Odat et al. [15] and Bidin and Nazar [16] regarding the flow and heat transfer of viscous fluid past an exponential stretching sheet. Khan and Sanjayanand [17], and Khan [18] have extended the work of Elbashbeshy [11] to the flow and heat transfer of a thermodynamically compatible second grade fluid. In a subsequent study (see Ref. [19]), they have added the mass transfer aspect.

In this work, the steady laminar flow and heat transfer of an electrically conducting third grade fluid over an exponentially

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stretching sheet with partial slip is considered. To the best of our knowledge, no attention has been given to the combined effects of partial slip and the magnetic field on the boundary layer flow and heat transfer of a third grade fluid past an exponentially stretching sheet.

2. Formulation of the problem

We consider the two-dimensional steady-state boundary layer flow and heat transfer of an incompressible, electrically conducting fluid of third grade over a stretching sheet. The constitutive equation of the thermodynamically compatible third grade fluid is given by [20]

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_3\left(tr\mathbf{A}_1^2\right)\mathbf{A}_1 \tag{1}$$

where $-p$ is the pressure, α_1 and α_2 are the normal stresses and β_3 is the material constant. $\mathbf{A}_1$ and $\mathbf{A}_2$ are the first two Rivlin–Ericksen tensors.

The sheet is coinciding with the plane $y = 0$ (see Fig. 1). The flow is assumed to be generated by stretching of the elastic boundary sheet from a slit with a large force such that the velocity of the boundary sheet is an exponential order of the flow directional coordinate x. The flow takes place in the upper half plane $y > 0$. A uniform magnetic field $\mathbf{B} = (0, B_0, 0)$ is imposed along the y-axis.

3. Flow analysis

For the physical problem, where the stretching of the boundary surface is assumed to be such that the flow directional velocity is of exponential order of the flow directional coordinate, the conventional no-slip boundary conditions are (see Ref. [11,17,21]):

$$\begin{aligned} &u = U_w(x) = U_0 \exp\left(\frac{x}{l}\right), \quad v = 0 \quad \text{at } y = 0, \\ &u = 0, \quad \text{as } y \to \infty. \end{aligned} \tag{2}$$

Here U_0 is the reference velocity and l is the reference length. The above exponential boundary condition is valid only when $x \ll l$, which occurs very near to the slit.

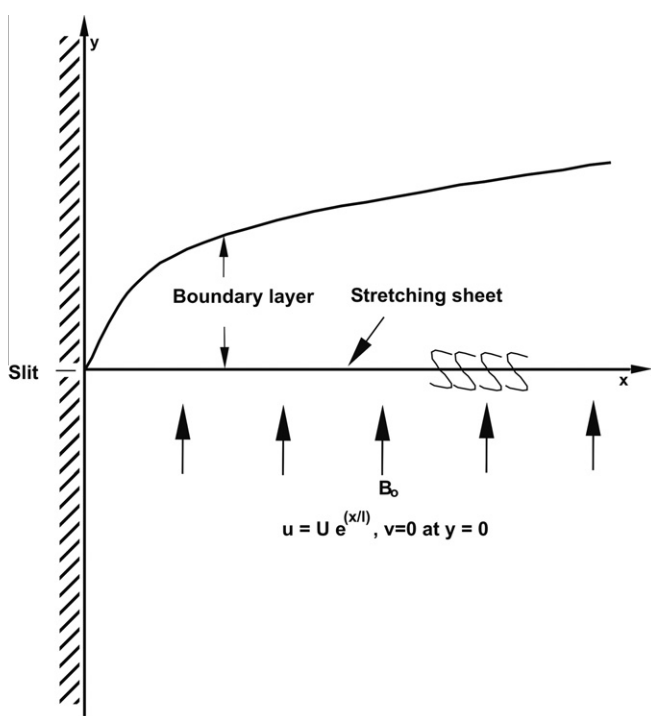

Fig. 1. Sketch of the flow past an exponentially stretching sheet.

Making the usual boundary layer approximations for the non-Newtonian third grade fluid (see Ref. [22]), the equations of continuity and motion can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$\begin{aligned} u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left[u\frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + 3\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right] \\ + \frac{2\alpha_2}{\rho}\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + \frac{6\beta_3}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u, \end{aligned} \tag{4}$$

where σ is the electrical conductivity of the fluid. The appropriate Navier's slip boundary conditions [1] of the velocity field are

$$\begin{aligned} &u - U_0\exp\left(\frac{x}{l}\right) = \lambda_1\nu\left[\frac{\partial u}{\partial y} + \frac{\alpha_1}{\mu}\left(2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + v\frac{\partial^2 u}{\partial y^2} + u\frac{\partial^2 u}{\partial x \partial y}\right) + 2\frac{\beta_3}{\mu}\left(\frac{\partial u}{\partial y}\right)^3\right], \ \text{at } y = 0, \\ &v(0) = 0, \ u \to 0 \ \text{as} \ y \to \infty. \end{aligned} \tag{5}$$

Eq. (4) can be rewritten in terms of a stream function $\psi(x, y)$ such that the continuity Eq. (3) is automatically satisfied. Hence

$$u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x}. \tag{6}$$

We choose the stream function to be

$$\psi(x, y) = \sqrt{2\nu l U_0}\,\varphi(\zeta)\exp\left(\frac{x}{2l}\right), \tag{7}$$

where

$$\zeta = y\sqrt{\frac{U_0}{2\nu l}}\exp\left(\frac{x}{2l}\right). \tag{8}$$

and φ is the dimensionless stream function. With the help of Eqs. (6)–(8), Eq. (4) gets reduced to,

$$\begin{aligned} \varphi''' - 2\varphi'^2 + \varphi\varphi'' + K\left[3\varphi'\varphi''' - \frac{1}{2}\varphi\varphi^{iv}\right] - \left(\frac{9}{2}K + 3L\right)\varphi''^2 \\ + 3\beta\varphi'''\varphi''^2 - 2M_n\varphi' = 0, \end{aligned} \tag{9}$$

where $K = \frac{\alpha_1 U_0}{\rho\nu l}, L = \frac{\alpha_2 U_0}{\rho\nu l}, \beta = \frac{\beta_3 U_0^3}{\rho\nu^2 l}, M_n = \frac{\sigma B_0^2 l}{\rho U_0}$ are the non-dimensional viscoelastic parameter, cross-viscous parameter, the third grade fluid parameter and the magnetic interaction parameter respectively.

The corresponding partial slip boundary conditions (5) on φ become

$$\begin{aligned} &\varphi(0) = 0, \quad \varphi'(0) - 1 = \lambda\varphi''(0)\left[1 + \frac{7}{2}K\varphi'(0) + \beta\varphi'^2(0)\right], \\ &\varphi'(\zeta) \to 0 \quad \text{as} \quad \zeta \to \infty, \end{aligned} \tag{10}$$

where the slip factor, $\lambda = \lambda_1\sqrt{\frac{U_0}{2\nu l}}$ represents the relative importance of the slip to viscous effects.

3.1. Heat transfer analysis

The thermal boundary layer equation for the thermodynamically compatible third grade fluid with viscous dissipation, work done due to deformation and Joule heating is

$$\begin{aligned} \rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \alpha_1\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right) \\ + 2\beta_3\left(\frac{\partial u}{\partial y}\right)^4 + \sigma B_0^2 u^2, \end{aligned} \tag{11}$$