International Journal of Heat and Mass Transfer 54 (2011) 5020–5029

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Effect of thermal conductivity and thickness of the walls in the convection of a viscoelastic Maxwell fluid layer

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ARTICLE INFO

Article history:
Received 21 October 2010
Received in revised form 5 July 2011
Accepted 5 July 2011
Available online 9 August 2011

Keywords:
Convection
Viscoelastic fluid
Thermal conductivity
Wall thickness

ABSTRACT

Results for the linear thermoconvective stability of a layer of viscoelastic Maxwell fluid are presented. The stability problem is characterized by taking into account the lower and upper wall thermal conductivities as well as their thicknesses. This allows more realistic theoretical boundary conditions. A system consisting of a horizontal infinite Maxwell fluid layer confined between two parallel walls perpendicular to gravity is considered. The critical Rayleigh number R_c, the frequency of oscillation ω_c and the wavenumber k_c were determined for fixed values of the relaxation time constant F and the Prandtl number Pr. The results are given for a range of wall thermal conductivities and thicknesses. Analytical and numerical solutions were calculated. Some unexpected results were found in comparison to those of the Newtonian fluid where the criticality curves become more unstable when the conductivities of the walls change from very good conductors to very bad conductors.

1. Introduction

Thermal convection of viscoelastic fluids may occur in many experimental set-ups and technological applications such as material processing, food and chemical industries. A particular area of research of growing interest where hydrodynamics of viscoelastic fluids is involved is that of the flow properties of biomolecules. In manipulation of biomolecules like DNA, for genome analysis and other applications, problems related to hydrodynamics arise and the theory of viscoelastic fluids can be used. Some efforts on this matter have been done since many years ago such as that of Bowen and Zimm [1] who determine some viscoelastic properties of DNA.

Thermal convection appearing in aqueous suspensions of DNA which behave as viscoelastic fluids (see [2] for example) is a very complex subject. This is due to a number of physical mechanisms that contribute or compete to set in convective cells in the suspension. Krishnan et al. [3] developed a device where Rayleigh convection is relevant to perform thermally activated chemical reactions such as polymerase chain reaction (PCR). In this case Krishnan et al. [3] proposed to replace the conventional thermocyclers by Rayleigh convection cells that make the device very simple and of easily assembly in any laboratory. Braun and Libchaber [4] proposed an efficient mechanism for trapping DNA in solution through the interaction of thermophoresis and thermoconvection. A study of the PCR in thermal convection for replication of DNA was conducted by Braun et al. [5] and by Braun [6]. In a more recent paper the interaction of thermophoresis and thermoconvection have been studied along with PCR for replication of DNA by Mast and Braun [7]. Theoretical advances on the hydrodynamics of this suspensions which exhibit viscoelastic behavior have been conducted by Sri Krishna [8] and Laroze et al. [9,10].

The aim of this paper is to study how the thermal properties and geometrical nature of the walls influence the hydrodynamic stability of a viscoelastic Maxwell fluid layer. The scenario we propose here has not been considered nor discussed before. The theory developed in this work may be significant to complement and understand the phenomena appearing in the applications.

The linear thermoconvective stability of a viscoelastic Maxwell fluid layer heated from below is investigated. The constitutive equation for the Maxwell fluid is used. It has a relaxation time that, when large, allows for important elastic properties. The physical problem investigated here is to understand the effect the thermal conductivity and thickness of the walls has on the instability. In the case of natural convection in a Newtonian fluid, this influence was investigated by Metcalfe and Behringer [11], Cerisier et al. [12] and Howle [13].

Natural convection in viscoelastic Maxwell fluids was first investigated by Vest and Arpaci [14] and in an Oldroyd fluid layer by Takashima [15]. In both papers, the temperature boundary conditions are of fixed temperature at the walls.

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doi:10.1016/j.ijheatmasstransfer.2011.07.016

Nomenclature

D_L	ratio of thicknesses (lower wall/fluid)
D_U	ratio of thicknesses (upper wall/fluid)
d_F	depth of fluid layer
d_L	thickness of lower wall
d_U	thickness of upper wall
F	relaxation time
g	acceleration due to gravity
K_L	thermal conductivity of the lower wall
K_F	thermal conductivity of the fluid
K_U	thermal conductivity of the upper wall
k	wavenumber
Pr	Prandtl number
R	Rayleigh number
T_F^*	dimensional fluid temperature profile
T_L^*	dimensional lower wall temperature profile
T_U^*	dimensional upper wall temperature profile
T_F	dimensionless fluid temperature profile
T_L	dimensionless lower wall temperature profile
T_U	dimensionless upper wall temperature profile
T_{BL}	temperature below the lower wall
T_{AU}	temperature above the upper wall
w	velocity perturbation
X_L	ratio of thermal conductivities (fluid/lower wall)
X_U	ratio of thermal conductivities (fluid/upper wall)
z^*	dimensional vertical coordinate
z	dimensionless vertical coordinate

Greek symbols

α	volumetric expansion coefficient of the fluid
θ	temperature perturbation
λ	relaxation time, s
κ	thermal diffusivity of the fluid, $cm^2\,s^{-1}$
v	kinematic viscosity, $cm^2\,s^{-1}$
ρ	fluid density, $g\,cm^{-3}$
ω	frequency of oscillation

Subscripts

c	critical value
L	lower
U	upper

The fixed heat flux boundary condition at the walls was investigated by Kolkka and Ierley [16] for natural convection of a viscoelastic Oldroyd fluid layer heated from below. Martínez-Mardones and Pérez-García [17] reported results for the stationary and oscillatory convection and a codimension-two point for the case of fixed temperature at the walls. Several advances in the study of convection in viscoelastic fluids have been made by Rosenblat [18], Park and Lee [19,20], Martínez–Mardones et al. [21], Dávalos-Orozco and Vázquez Luis [22], Martínez–Mardones et al. [23–26] and more recently by Li and Khayat [27]. Some other advances in coupled buoyancy and capillary thermoconvection in viscoelastic fluids have also been made by Dauby et al. [28], Lebon et al. [29] and Parmentier et al. [30]; and earlier for capillary thermoconvection alone by Getachew and Rosenblat [31].

The system consists of a horizontal fluid layer between two parallel walls which are perpendicular to gravity. The system is characterized by nondimensional parameters such as: the Rayleigh number R, the Prandtl number Pr, the relaxation time constant F, the thermal conductivity ratios of the fluid to the lower and upper walls (X_L,X_U), the thickness ratios of the lower and upper walls respect to that of the fluid layer (D_L,D_U), the frequency of oscillation ω and the perturbation wavenumber k. The results presented here are of great importance because the physical and geometrical properties of the walls are taken into account. This allows the theory to better simulate the real experimental conditions.

The paper is organized as follows. In Section 2 the formulation of the problem is given including the governing equations, boundary conditions as well as nondimensional parameters of the problem. In Section 3 a solution to the eigenvalue problem using analytical techniques and numerical methods is presented. Finally, a discussion of the results is presented in the last section.

2. Formulation of the problem

Consider the natural convection in a viscoelastic Maxwell fluid layer confined between two infinite horizontal walls perpendicular to gravity. The lower and upper walls have thicknesses (d_L,d_U) and thermal conductivities (K_L,K_U), respectively. The upper surface of the lower wall and lower surface of the upper wall are located at $z = 0$ and $z = 1$, respectively. The fluid layer has density ρ, dynamic viscosity ρv (with v being the kinematic viscosity), thermal conductivity K_F and thickness d_F.

The equations of momentum and energy of the incompressible Maxwell fluid are linearized and perturbed (see [14,15]). Next, the rotational operation is taken twice in the momentum equation to obtain the following system of coupled equations for the perturbations

$$\left(1+F\frac{\partial}{\partial t}\right)\left(\frac{1}{Pr}\frac{\partial}{\partial t}\nabla^2 w - R\nabla_\perp^2\theta\right) = \nabla^4 w \tag{1}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)\theta = w \tag{2}$$

where w is the fluid velocity and θ is the temperature. The dimensionless parameters in Eqs. (1) and (2) are $F = \lambda\kappa/d_F^2$ the relaxation time, $Pr = v/\kappa$ the Prandtl number and $R = \alpha g d_F^3(T_{BL} - T_{AU})/[v\kappa(1 + D_U X_U + D_L X_L)]$ the Rayleigh number. Dimensionless variables are obtained by using the following scales: d_F for length, d_F^2/κ for time, $(T_{BL} - T_{AU})/(1 + D_U X_U + D_L X_L)$ for temperature and κ/d_F for velocity. Notice that $T_{BL} > T_{AU}$.

In the basic state there is no motion in the fluid and the heat transport is only by conduction. Before perturbation, the main temperature profiles for the fluid and walls are calculated from the linear stationary heat diffusion equation $d^2T/dz^{*2} = 0$. These dimensional solutions satisfy the following thermal boundary conditions. The temperature is constant over the outer surface of each wall, that is, $T_L^* = T_{BL}$ at $z^* = -d_L$ and $T_U^* = T_{AU}$ at $z^* = d_F + d_U$. They satisfy the continuity of temperature and heat flux at the interface of the fluid with each wall at $z^* = 0$ and d_F, respectively. The solutions, in dimensional form, are:

$$T_F^* = -\frac{(T_{BL}-T_{AU})z^*}{d_F(1+D_U X_U + D_L X_L)} + T_{BL} - \frac{(T_{BL}-T_{AU})D_L X_L}{(1+D_U X_U + D_L X_L)} \tag{3}$$

$$T_L^* = -\frac{(T_{BL}-T_{AU})X_L z^*}{d_F(1+D_U X_U + D_L X_L)} + T_{BL} - \frac{(T_{BL}-T_{AU})D_L X_L}{(1+D_U X_U + D_L X_L)} \tag{4}$$

$$T_U^* = -\frac{(T_{BL}-T_{AU})X_U z^*}{d_F(1+D_U X_U + D_L X_L)} + T_{BL} - \frac{(T_{BL}-T_{AU})(1+D_L X_L - X_U)}{(1+D_U X_U + D_L X_L)} \tag{5}$$

Lets assume that $T_i = (T_i^* - T_{AU})(1 + D_U X_U + D_L X_L)/(T_{BL} - T_{AU})$, where the subscript i stands for (F,L,U). Then, in nondimensional form they can be rewritten as