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From natural to mixed convection in horizontal and differentially heated annular ducts: Linear stability analysis

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ABSTRACT

Linear stability analysis of a fully developed mixed convection flow of air in an annular horizontal duct is numerically investigated for the radius ratio $R = 1.2$, a Péclet and a Rayleigh number less than 200 and 6000, respectively. An iterative method is developed to enable the convergence of the dimensionless parameters to their marginal values at the transition. New mixed convection flows are highlighted that are highly correlated with those obtained in natural convection problems under the assumption of two dimensionality. The synthesis of our results on the transitions permits us to build the map of stability for the steady and established mixed convection flows and clearly shows the occurrence of multiplicity of solutions for some couples of Rayleigh and Péclet numbers.

1. Introduction

Natural, forced and mixed convection in horizontal annuli is a fundamental issue of interest and has been extensively studied. This interest stems from the wide range of related engineering applications such as thermal energy storage systems, heat exchangers, transmission cables, solar collectors, etc.

Natural convection in differentially heated horizontal annuli inspired numerous studies because of the role of curvature on the birth of thermal instabilities. Although early experimental work dates from 1931 (Beckmann [1]), it took 40 years to have a qualitative description of flows depending on the Grashof number and radius ratio (Grigull and Hauf [2], Powe et al. [3]). With the increase in computational resources, numerous numerical simulations were carried out, but mainly under the assumption of two-dimensional flows, invariant in the axial direction. These studies show that two-dimensional flow, which develops in the form of two large symmetrical and crescent-shaped cells, undergoes a Rayleigh–Bénard instability with the increase in the Rayleigh number, for radius ratio in the range $1.2 \leqslant R \leqslant 2$ (see Petrone et al. [4] for example). The supercritical flow pattern is then made of one or two pairs of additional convection rolls located at the top of the annulus, thereby enhancing heat transfer rate between the cylinder walls. However, these two-dimensional flows turn out to be unstable with respect to three-dimensional perturbations [5–8]. A critical review of buoyancy-induced flow transitions in horizontal annuli can be found in a recent paper by Angeli et al. [9].

Forced convection, and to a lesser extent mixed convection, have been the subject of many analytical, experimental and numerical investigations, concerning both the entrance regions (dynamical and thermal) and the heat transfer for fully developed flows [10]. Graetz [11] (1883), Nusselt [12] (1910) and later on Lévêque [13] (1928) were interested in the issue of the developing thermal regime for a fluid flowing in the laminar established regime in a pipe whose walls were maintained at uniform temperature. In this model, the axial diffusion is neglected, such an assumption is justified when the Péclet number is sufficiently high (Pe > 100). Based on similar assumptions, the works of Lundberg et al. [14] and Shah and London [15] provided a comprehensive study on the establishment of thermal regime in an annular duct for several combinations of flow conditions and temperature applied at pipe walls. With similar assumptions, Kakaç and Yücel [16] studied the laminar flow heat transfer in annuli with simultaneous development of velocity and temperature fields. For low values of Péclet number, both axial diffusion [17,18] and free convection [19,20] become not negligible in respect of the establishment length value, which is also strongly affected by thermal conditions applied at the walls. Amongst the papers dealing with the entrance regions, a few are devoted to experimental investigations (see for example the recent paper of Mohammed et al. [21] and references herein). Finally, to our best knowledge, few numerical studies were focused on the influence of natural convection in dynamically and thermally fully developed flows in annular ducts for very low Reynolds number values, and only for large radii ratios [22].

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Nomenclature

$\boldsymbol{e}_3$	vertical unit vector pointing upwards
f	$=\eta r+1$
g	gravity acceleration (m/s^2)
i	pure imaginary number
k	real wavenumber
p	pressure
Pe	Péclet number, $=\frac{\bar{w}^*(r_o^\star-r_i^\star)}{\alpha}$
Pr	Prandtl number, $=\frac{\nu}{\alpha}$
r	reduced radial coordinate, $=\frac{r^\star-r_i^\star}{r_o^\star-r_i^\star}$
(r,θ,z)	cylindrical coordinates
R	radius ratio, $=r_o^\star/r_i^\star$
Ra	Rayleigh number, $=\frac{g\beta(T^*(r_i^\star)-T^*(r_o^\star))(r_o^\star-r_i^\star)^3}{\nu\alpha}$
$\widehat{\mathrm{Ra}}(k)$	critical Rayleigh number function of the k
$r_i^\star$	inner radius (m)
$r_o^\star$	outer radius (m)
T	temperature
$\boldsymbol{v}$	velocity vector, $=u\boldsymbol{e}_r+v\boldsymbol{e}_\theta+w\boldsymbol{e}_z$
$\bar{w}$	mean axial velocity

Subscripts

0	steady basic solution
c	threshold value
k	component in the Fourier space
k,λ	component in the Fourier and Laplace spaces

Superscripts

1st	first branch of solutions
2nd	second branch of solutions
$*$	dimensional variable

Greek symbols

α	thermal diffusivity (m^2/s)
β	expansion coefficient (K^{-1})
$\delta\boldsymbol{v}$	velocity vector for the perturbation, $=\delta u\boldsymbol{e}_r+\delta v\boldsymbol{e}_\theta+\delta w\boldsymbol{e}_z$
ΔP	constant axial pressure gradient
η	relative annular gap, $=R-1$
λ	complex eigenvalue, $=\lambda_r+i\lambda_i$
λ_r	growth rate
λ_r^M	maximum growth rate
λ_i	pulsation
λ_i^M	pulsation of the complex eigenvalue having λ_r^M as real part
ν	kinematic viscosity (m^2/s)
ρ	density (kg/m^3)

Despite these numerous studies, there are still many aspects that need to be explored or thorough, especially concerning the effects of an axial flow on the multicellular secondary flows induced by the buoyancy force in narrow annular spaces. To this aim, a linear stability analysis of the fully developed flow is performed for air flowing in an annular pipe of fixed radius ratio $R=1.2$. The rest of the paper is structured as follows. Section 2 is devoted to the presentation of the governing equations for the basic flow and the perturbed states. A numerical method, suitable for calculating the transition thresholds in a plane of Rayleigh number and Péclet number, is presented. It is built around an iterative method, coupling the calculation of the basic steady flow and determination of the dominant spectrum of the linearized problem. The iterative process, involving the wavenumber and Rayleigh number, is based on approximate Newton methods for which the derivatives are substituted by simple algebraic relations. Section 3 emphasizes the close link between our previous works about pure free convection and the nature of the flow that develops in the cross sections of fluid flow in mixed convection. The sensitivity of the critical Rayleigh number is studied as a function of the Péclet number, and it is shown that topologies which are linearly unstable in natural convection turn out to be stable in mixed convection. In particular we show that multiple solutions are simultaneously stable for certain ranges of the couple (Pe, Ra). Finally, a conclusion is drawn that highlights the main issues of this work.

2. Equations

2.1. Physical model

The horizontal annular pipe is confined by two co-axial and infinite cylinders of radii $r_i^\star$ and $r_o^\star>r_i^\star$ (Fig. 1). The temperature of the inner and outer cylinders is kept constant such that $T^\star(r_o^\star)<T^\star(r_i^\star)$. The fluid flow is assumed incompressible with constant physical properties except the density in the buoyancy term. The axial coordinate is scaled by the annulus gap $r_o^\star-r_i^\star$, the velocity components by the mean axial velocity $\bar{w}^*$, the dynamical pressure by $\rho(\bar{w}^*)^2$ and the time by $(r_o^\star-r_i^\star)/\bar{w}^*$. We also introduce the dimensionless temperature difference $T=(T^\star-T_r^\star)/(T^\star(r_i^\star)-T^\star(r_o^\star))$ with $T_r^\star=(T^\star(r_i^\star)+T^\star(r_o^\star))/2$, and the reduced radial coordinate $r=(r^\star-r_i^\star)/(r_o^\star-r_i^\star)$.

To shorten the writing of equations presented in this article, and to emphasize the role of two-dimensional flows that develop in planes transverse to the axis of the cylinders, the partial derivative operators have been split into an implicit part coupling the radial and azimuthal directions, and symbolically represented by "$\boldsymbol{\nabla}_{2d}\cdot$", "$\overline{\boldsymbol{\nabla}_{2d}}\cdot$", "$\nabla_{2d}^2$", "$\overline{\nabla_{2d}^2}$" and "$\boldsymbol{\nabla}_{2d}$", and an explicit part that deals only with the axial derivatives. Thus, by combining the radial and azimuthal components of the momentum equation into a single vectorial relation (see Eq. (1b)), we obtain the three-dimensional Navier–Stokes and energy equations as follows:

$$\boldsymbol{\nabla}_{2d}\cdot\boldsymbol{v}+\frac{\partial(fw)}{\partial z}=0 \tag{1a}$$

$$\frac{\partial}{\partial t}(fu)\boldsymbol{e}_r+\frac{\partial}{\partial t}(fv)\boldsymbol{e}_\theta+\overline{\boldsymbol{\nabla}_{2d}}\cdot(\boldsymbol{v}\otimes\boldsymbol{v})+\frac{\partial(fwu)}{\partial z}\boldsymbol{e}_r+\frac{\partial(fwv)}{\partial z}\boldsymbol{e}_\theta=-\boldsymbol{\nabla}_{2d}p+\frac{\mathrm{RaPr}}{\mathrm{Pe}^2}fT\boldsymbol{e}_3+\frac{\mathrm{Pr}}{\mathrm{Pe}}\left(\overline{\nabla_{2d}^2}\boldsymbol{v}+\frac{\partial}{\partial z}\left(f\frac{\partial u}{\partial z}\right)\boldsymbol{e}_r+\frac{\partial}{\partial z}\left(f\frac{\partial v}{\partial z}\right)\boldsymbol{e}_\theta\right) \tag{1b}$$

$$\frac{\partial}{\partial t}(fw)+\boldsymbol{\nabla}_{2d}\cdot(w\boldsymbol{v})+\frac{\partial(fw^2)}{\partial z}=-f\frac{\partial p}{\partial z}+\frac{\mathrm{Pr}}{\mathrm{Pe}}\left(\nabla_{2d}^2+\frac{\partial}{\partial z}\left(f\frac{\partial}{\partial z}\right)\right)w \tag{1c}$$

$$\frac{\partial}{\partial t}(fT)+\boldsymbol{\nabla}_{2d}\cdot(T\boldsymbol{v})+\frac{\partial(fTw)}{\partial z}=\frac{1}{\mathrm{Pe}}\left(\nabla_{2d}^2+\frac{\partial}{\partial z}\left(f\frac{\partial}{\partial z}\right)\right)T \tag{1d}$$

with the following definitions for the two-dimensional operators:

- Divergence of the vector field $X\boldsymbol{v}$

$$\boldsymbol{\nabla}_{2d}\cdot(X\boldsymbol{v})=\frac{\partial(fXu)}{\partial r}+\frac{\partial(\eta Xv)}{\partial\theta} \tag{2a}$$

- Divergence of the tensorial field $\boldsymbol{v}\otimes\boldsymbol{v}$

$$\overline{\boldsymbol{\nabla}_{2d}}\cdot(\boldsymbol{v}\otimes\boldsymbol{v})=(\boldsymbol{\nabla}_{2d}\cdot(u\boldsymbol{v})-\eta v^2)\boldsymbol{e}_r+(\boldsymbol{\nabla}_{2d}\cdot(v\boldsymbol{v})+\eta uv)\boldsymbol{e}_\theta \tag{2b}$$