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Effect of wall properties on the peristaltic flow of a third grade fluid in a curved channel with heat and mass transfer

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ABSTRACT

Analysis has been carried out to study the heat and mass transfer effects on the peristaltic flow in a curved channel with compliant walls. Firstly, mathematical modelling is performed and then solution is obtained under the assumptions of long wavelength and low Reynolds number. Stream function, temperature and concentration fields are derived. The effects of emerging parameters in the obtained solutions are discussed.

1. Introduction

Peristaltic transport appears in several biological systems, for example smooth muscle tubes for instance lower intestine, cervical canal, gastrointestinal tract, lymphatic vessels and small blood vessels. The roller and finger pumps also work under this principle. In nuclear industry, toxic liquid can be transported by such mechanism in order to avoid contamination of the outside environment. Vast literature is available on this topic for viscous and the non-Newtonian fluids. Mekheimer and elmaboud [1] studied the peristaltic motion of couple stress fluid in an annulus. Influence of induced magnetic field on peristaltic transport of couple stress fluid in a planar channel is discussed by Mekheimer [2]. Rao and Mishra [3] studied the peristaltic flow of power-law fluid in porous tubes. Ebaid [4] investigated the peristaltic flow of viscous fluid in an asymmetric channel with magnetic field and wall slip conditions. Haroun [5] discussed the peristaltic flow of third order fluid in an asymmetric channel. In another investigation, Haroun [6] analyzed the peristaltic flow of fourth grade fluid in an inclined channel. Wang et al. [7] studied the peristaltic transport of third grade fluid in a tube subject to the slip conditions. Kothandapani and Srinivas [8] discussed the MHD peristaltic flow of Jeffrey fluid in an asymmetric channel. Peristaltic flow of Newtonian fluid in an inclined asymmetric channel through porous medium is discussed by Kothandapani and Srinivas [9]. Hayat et al. [10] studied the slip effect on MHD peristaltic flow of viscous fluid with variable viscosity. Hayat et al. [11] examined the peristaltic flow of Johnson–Segalman fluid in an asymmetric channel. Slip effects on peristaltic flow in a porous medium is also examined by Hayat et al. [12]. The peristaltic flow of Maxwell fluid in an asymmetric channel is studied by Hayat et al. [13]. Slip effect on peristaltic flow of third order fluid in an asymmetric channel is presented by Hayat et al. [14].

In existing literature, the reasonable attention is given to the peristaltic flows in a channel having compliant walls. Mittra and Prasad [15] analyzed the effects of wall properties on peristalsis. Davies and Carpenter [16] analyzed the stability of plane channel flow between compliant walls. Haroun [17] studied the compliant wall effects on peristalsis in an asymmetric channel. Radhakrishnamacharya and Srinivasulu [18] studied the peristaltic transport with heat transfer effects in a symmetric channel with flexible walls. Muthu et al. [19] discussed the peristaltic flow of micropolar fluid in a circular cylindrical tube with flexible walls. Elnaby and Haroun [20] studied the effect of wall properties on peristaltic transport of a viscous fluid. Hayat et al. [21,22] have seen the influence of compliant wall properties on the peristaltic flow of Johnson–Segalman and Jeffery fluids in a channel. Peristaltic transport of Maxwell fluid in a channel with compliant walls is considered by Ali et al. [23]. Kothandapani and Srinivas [24] studied the effects of wall properties on MHD peristaltic flow of viscous fluid in a channel with porous medium. This work is extended by Srinivas et al. [25] in the presence of slip effects. Srinivas and Kothandapani [26] have also discussed the compliant wall effects on MHD peristaltic flow through a porous space.

Peristaltic flow with heat transfer has many applications in the biomedical sciences. Heat transfer involves many complicated processes in tissues such as heat conduction in tissues, heat con-

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vection due to the blood flow from the pores of the tissues and radiation between surface and its environment. There is obvious involvement of mass transfer in all such processes. The processes of hemodialysis and oxygenation have been visualized by considering peristaltic flows with heat transfer. When simultaneous effects of heat and mass transfer are considered, the complicated relationships occur between the fluxes and the driving potentials. The energy flux is induced by temperature gradient as well as composition gradients and mass flux can be produced by temperature gradient (i.e. Soret effect). Mass transfer phenomenon is important in the diffusion process such as the nutrients diffuse out from the blood to the neighboring tissues. Research on bioheat transfer discusses the heat and mass transfer in organisms. Investigation of heat and mass transfer in peristalsis has been considered by some researchers. Radhakrishnamacharya and Murty [27] studied the heat transfer on the peristaltic transport in a non-uniform channel. Srinivas and Kothandapani [28] investigated the peristaltic transport in an asymmetric channel with heat transfer. Vajravelu et al. [29] reported the peristaltic flow and heat transfer in a vertical annulus under long wavelength approximation. The influence of heat transfer and magnetic field on peristaltic transport of Newtonian fluid in a vertical annulus has been discussed by Mekheimer and elmaboud [30]. Influence of heat and mass transfer on peristaltic flow of third order fluid in a diverging tube is discussed by Nadeem et al. [31]. Hayat and Hina [32] analyzed the effects of heat and mass transfer on the peristaltic transport of Maxwell fluid in a porous channel with compliant walls. Two-dimensional peristaltic motion in a curved channel has been firstly addressed by Sato et al. [33]. Ali et al. [34] discussed the peristaltic flow of viscous fluid in a curved channel. Heat transfer analysis of peristaltic flow in a curved channel is analyzed by Ali et al. [35]. Ali et al. [36] also examined the peristaltic flow of a third grade fluid in a curved channel.

It is noticed from the available literature that no analysis has been made yet for the peristaltic flow of a non-Newtonian fluid in a curved channel with compliant walls and heat and mass transfer. A subclass of differential type non-Newtonian model namely the third grade fluid is considered in this paper. It is now established fact that majority of the physiological fluids are non-Newtonian in character. The third grade has an ability to describe the shear thinning/shear thickening properties even in the steady situation. The aim of current attempt is two fold. Firstly to extend the analysis of Ref. [35] by considering heat and mass transfer effects. Secondly, to discuss the analysis of the wall properties. In this work, series solutions are demonstrated for small Deborah number. The derived solutions are plotted and analyzed in detail.

2. Mathematical modelling

We consider the flow of an incompressible third grade fluid in a curved channel of radius R^* and uniform thickness $2d_1$ coiled in a circle with centre O (see Fig. 1). The flow is in the axial direction x and r is radial direction, u and v are the components of velocity in the axial and radial directions respectively.

The wave shapes of the walls are

$$r = \pm\eta(x,t) = \pm\left[d_1 + a\sin\frac{2\pi}{\lambda}(x-ct)\right], \tag{1}$$

in which c is the wave speed and a and λ are the wave amplitude and wavelength respectively. The equations which can govern the flow can be written as

$$\frac{\partial v}{\partial r} + \frac{R^*}{r+R^*}\frac{\partial u}{\partial x} + \frac{v}{r+R^*} = 0, \tag{2}$$

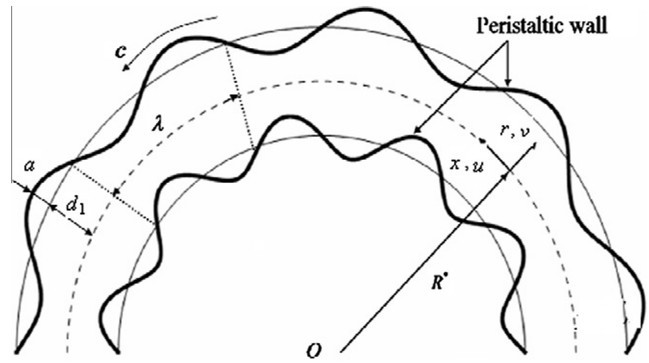

Fig. 1. Physical model.

$$\rho\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial v}{\partial x} - \frac{u^2}{r+R^*}\right] = -\frac{\partial p}{\partial r} + \frac{1}{r+R^*}\frac{\partial}{\partial r}\{(r+R^*)S_{rr}\} + \frac{R^*}{r+R^*}\frac{\partial S_{xr}}{\partial x} - \frac{S_{xx}}{r+R^*}, \tag{3}$$

$$\rho\left[\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial u}{\partial x} + \frac{uv}{r+R^*}\right] = \frac{1}{(r+R^*)^2}\frac{\partial}{\partial r}\left\{(r+R^*)^2 S_{rx}\right\} + \frac{R^*}{r+R^*}\frac{\partial S_{xx}}{\partial x} - \frac{R^*}{r+R^*}\frac{\partial p}{\partial x}, \tag{4}$$

$$\rho C_p\left[\frac{\partial}{\partial t} + v\frac{\partial}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial}{\partial x}\right]T = \kappa\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R^*}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right] + (S_{rr}-S_{xx})\frac{\partial v}{\partial r} + S_{xr}\left(\frac{\partial u}{\partial r} + \frac{R^*}{r+R^*}\frac{\partial v}{\partial x} - \frac{u}{r+k}\right), \tag{5}$$

$$\left[\frac{\partial}{\partial t} + v\frac{\partial}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial}{\partial x}\right]C = D\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r+R^*}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial x^2}\right]C + \frac{DK_T}{T_m}\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r+R^*}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}\right], \tag{6}$$

where the extra stress tensor in a third grade fluid is defined by the following expression

$$\mathbf{S} = \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta\left(tr\mathbf{A}_1^2\right)\mathbf{A}_1$$

in which the material parameters μ, α_i ($i = 1,2$) and β must satisfy

$$\beta \geqslant 0, \quad \alpha_1 \geqslant 0, \quad |\alpha_1+\alpha_2| \leqslant \sqrt{24\mu\beta}$$

and the Rivlin-Ericksen tensors are

$$\mathbf{A}_1 = grad\mathbf{V} + (grad\mathbf{V})^T, \quad \mathbf{A}_2 = \frac{d\mathbf{A}_1}{dt} + \mathbf{A}_1(grad\mathbf{V}) + (grad\mathbf{V})^T\mathbf{A}_1,$$

where d/dt is the material time differentiation. The boundary conditions are

$$u = 0, \quad T = T_0, \quad C = C_0 \text{ at } r = \pm\eta,$$

$$R^*\left[-\tau\frac{\partial^3}{\partial x^3} + m\frac{\partial^3}{\partial x\partial t^2} + d\frac{\partial^2}{\partial t\partial x}\right]\eta = \frac{1}{(r+R^*)}\frac{\partial}{\partial r}\{(r+R^*)^2 S_{rx}\} + \frac{\partial S_{xx}}{\partial x} - \rho(r+R^*)\times\left[\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial r} + \frac{R^* u}{r+R^*}\frac{\partial u}{\partial x} + \frac{uv}{r+R^*}\right] \text{at} r = \pm\eta. \tag{7}$$

Here T_0 and C_0 indicate the temperature and the mass concentration at both the walls of channel respectively, p the pressure, μ the kinematic viscosity, $\rho$the density, R^* the curvature parameter, C_p the specific heat at constant volume, T the temperature of fluid, C the