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Analysis of the 1D heat conduction problem for a single fin with temperature dependent heat transfer coefficient – Part II. Optimum characteristics of straight plate and cylindrical pin fins

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Abstract

An exact hypergeometric implicit solution of the 1D steady-state heat conduction problem for a straight fin of constant cross-section is used to calculate the dependence of the main dimensionless fin parameters, specifically, base thermal conductance G and thermo-geometrical fin parameter N on T_e, the ratio of the fin tip to fin base temperature excesses. The straight plate fin (SPF) and cylindrical pin fin (CPF) with an insulated tip (INT) and non-insulated tip (NINT) are optimized. The local heat transfer coefficient (HTC) is assumed to vary as power function of the local fin excess temperature with arbitrary value of exponent n in the range of $-0.5 \leqslant n \leqslant 5$. Every curve from G vs T_e set at given n for a fin with an INT is shown to have a single global maximum $G = G^*_{opt}$ at $T_e = T^*_{e,opt}$ and corresponding $N = N^*_{opt}$, i.e. the main optimum parameters depend only on exponent n. Every curve from G vs T_e set for a fin with a NINT depends, in addition, on the complex fin tip parameter B_ω. These curves have the local maximum and minimum points. As B_ω increases these points approach each other and at $B_\omega = B^{**}$ merge. The corresponding curve G vs T_e has the only inflection point. The main optimum parameters of a fin with an INT and inflection point parameters of a fin with a NINT are approximated by general homographic function of n. Each main optimum parameter of a fin with a NINT is expressed as a product of the corresponding parameter of this fin with an INT and a correction factor approximated by the generalized closed-form formula. The results of the study are presented in form of dimensionless explicit relations, tables and plots which are well suited for the thermal design of optimum fins.

Keywords: Single fin (SPF and CPF); Insulated and non-insulated tip; Temperature dependent HTC; Optimization procedure; Explicit closed-form formulae

1. Introduction

Authors of the fundamental book "Extended Surface Heat Transfer" [1] have pointed out that there are three types of optimizations that pertain to extended surface design and analysis. The first type of optimization involves for longitudinal fins, radial fins, and spines the finding of the profile shape that yields maximum heat flow (or thermal conductance in terms of present study) for a specified mass of a fin (the direct optimization problem) or minimum mass for a specified heat flow or thermal conductance of a fin (the inverse optimization problem). The second type of optimization is, in essence, a variant of the first one for a fin with given shape of profile. For example, optimization of longitudinal fins of constant thickness or the straight plate fins (SPF) and cylindrical spines or cylindrical pin fins (CPF) will be considered in the present paper. The optimization problems of this type are considered in the most publications in available technical literature. The third type of optimization consists in application of the mentioned optimization types to an array of fins in which each fin is operating in an optimum manner.

The paper by Aziz [2] contain a review on optimum dimensions of extended surfaces (mostly, single fins of different shape) losing heat by pure convection to the

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Nomenclature

a, a_e given constants in the heat transfer equation for the lateral surfaces and tip surface of a fin ($\mathrm{W\,m^{-2}\,K^{-(n+1)}}$)
a_p profile area of the SPF ($\mathrm{m^2}$)
A fin cross-sectional area ($\mathrm{m^2}$)
$A_p, \widehat{A}_p$ dimensionless profile area of the SPF (whole and reduced), $A_p = a_p(h_b/k)^2$; $\widehat{A}_p = A_p/2$
B, B_ω general designation for given Biot number or fin base conductance G_z (or G_c) of the SPF (or CPF), $B_\omega = B \cdot f(\omega)$
Bi, Bi_l, Bi_a, Bi_v dimensionless Biot numbers based on the thickness (radius), height, profile area (or volume) of the SPF (or CPF), respectively
E_f extension factor of the total fin heat transfer surface, F/A
$f(\omega)$ power function of the tip heat transfer ratio with different value of p for every given parameter B of the SPF (or CPF), $f(\omega) = \omega^p$
F whole heat transfer area of a fin ($\mathrm{m^2}$)
$C_{i,i=1,3}$ numerical coefficients of the homographic function in Eq. (16)
g_b fin base thermal conductance, Q_b/ϑ_b ($\mathrm{W\,K^{-1}}$)
g_l thermal conductance of a fin with insulated lateral surfaces, kA/l ($\mathrm{W\,K^{-1}}$)
G dimensionless thermal conductance of the SPF (or CPF), $G = \widehat{G}_z/(Bi_a^2/2)^{1/3}$ (or $G = \widehat{G}_c/[Bi_v^3/(4\pi)]^{3/5}$), respectively
G_b dimensionless fin base thermal conductance, g_b/g_l
$G_c, \widehat{G}_c$ dimensionless total and reduced thermal conductance of the CPF, $G_c = g_b(h_b/k^2)$, $\widehat{G}_c = G_c/(4\pi)$
G_{cV} dimensionless thermal conductance of the CPF per unit volume, G_c/V
$G_z, \widehat{G}_z$ dimensionless total and reduced thermal conductance of the SPF, $G_z = g_b/(zk)$; $\widehat{G}_z = G_z/2$
G_{zA_p} dimensionless thermal conductance of the SPF per unit profile area, G_z/A_p
h, h_e heat transfer coefficient for the lateral and tip surfaces of a fin ($\mathrm{W\,m^{-2}\,K^{-2}}$)
K fin augmentation factor (effectiveness), g_b/g_p
k thermal conductivity of the fin material ($\mathrm{W\,m^{-1}\,K^{-1}}$)
l fin height (m)
N dimensionless thermo-geometrical fin parameter, $l\sqrt{h_b P/(kA)}$
n given exponent in the heat transfer equation for a fin
P perimeter of the fin cross-section (m)
r radius of the CPF (m)
Q_{bz}, Q_b heat flow dissipated by the SPF (or CPF), ($\mathrm{W\,m^{-1}}$, (W))
t fin temperature (K)
T dimensionless fin temperature excess, ϑ/ϑ_b
T_e dimensionless fin tip temperature excess, ϑ_e/ϑ_b
v volume of the CPF ($\mathrm{m^3}$)
$V, \widehat{V}$ dimensionless total and reduced volume of the CPF, $V = v(h_b/k)^3$, $\widehat{V} = V/(4\pi)$
x space coordinate (m)
X dimensionless space coordinate, x/l
Y general denotation of the dimensionless dependent variable that have to be optimized
z width of the SPF (m)

Greek symbols
$\beta_{\omega,B}$ dimensionless complex parameter of heat transfer on the fin tip, B_ω/B^{**}
δ fin thickness (m)
ϑ local temperature excess of a fin over the ambient medium, $t - t_a$ (K)
$\xi_{Y,B}$ correction factor to determine the required optimum value, Y_{opt}/Y^*_{opt}
ψ fin aspect ratio (fin height to half-thickness or half-radius ratio), Bi_l/Bi
ω ratio of heat transfer coefficients on the tip and lateral surfaces of a fin, $h_e/h_{x=0}$

Subscripts and superscripts
b, e refer to the fin base and fin tip (for $X = 1$ and $X = 0$, respectively)
opt refer to optimum values
* refer to the fin with an insulated tip
** refer to the inflection point of G vs T_e curve for the fin with a non-insulated tip at $\omega = 1$ and maximum allowable B value (B^{**})

surroundings. The review covers straight (longitudinal) fins, annular (radial) fins and spines of different profile shapes. The optimum dimensions for each shape are given both in terms of the given volume of material as well as in terms of the given heat dissipation. The effects of tip heat loss, variable heat transfer coefficient, internal heat generation, and temperature dependent thermal conductivity of fin material on the optimum dimensions have been discussed.

The direct analytical solution of the optimization problem for a single CPF with an insulated tip and uniform heat transfer coefficient on the cylindrical surface is obtained by Sonn and Bar-Cohen in [3]. Authors have expressed the heat flow Q_b dissipated by a cylindrical spine in terms of its diameter d. The values of fin volume and fin base temperature excess ϑ_b are given. Thermal conductivity of the fin material k and heat transfer coefficient h over the whole fin surface are assumed to be constant. Differentiating Q_b