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Optimum length of tubes for heat transfer in turbulent flow at constant wall temperature

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Abstract

Maximum heat transfer per cross-sectional area of a tube with smooth wall in turbulent flow at constant wall temperature is determined for a given pressure loss. The dimensionless tube length is determined dependent on the pressure Reynolds number, Prandtl number and inlet local pressure loss coefficient. Limiting cases for short and long tubes are separately investigated. Semi-empirical equations are derived for both optimum dimensionless tube length and dimensionless maximum heat flow per cross-sectional area using numerically obtained values with a maximum deviation of ±6.6% and with a RMSE of 3.5%. The results can also be applied to the channels with non-circular cross-sectional area.

Keywords: Optimum length; Turbulent flow; Constant wall temperature; Forced convection

1. Introduction

Heat exchangers should be constructed as compact as possible. Therefore, they should be designed so that optimum heat transfer occurs. Optimum conditions can occur both for natural and forced convective heat transfer. Different optimum heat transfer conditions are described by Middleman [1].

In many applications heat is transferred by natural convection. Optimum spacing in vertical parallel plates for natural convection is presented by Bar-Cohen and Rohsenow [2] for isothermal symmetric and asymmetric heating and isoflux heating boundary conditions. This problem is investigated numerically by Morrone et al. [3] considering second derivatives in flow direction. Optimum spacing between horizontal cylinders is investigated analytically and numerically by Bejan et al. [4]. Optimum distance between vertical fins by natural convection is determined analytically by Vollaro et al. [5]. Optimum conditions for vertical ducts of arbitrary cross-sectional area are obtained using analytical and experimental results by Yılmaz and Oğulata [6].

Forced convection is the most commonly encountered mode of heat transfer. Bejan and Sciubba [7] and Campo [8] determined optimal plate channel spacing for forced convection. Fowler et al. [9] investigated both numerically and experimentally optimal placing of staggered plates in forced convection. Matos et al. [10] calculated numerically heat transfer around staggered circular and elliptical tubes with constant surface temperature in forced convection. They determined optimal spacing between the circular or elliptical surfaces as a function of Reynolds number. Yılmaz et al. [11] presented optimum dimensions of ducts for laminar flow at constant wall temperature.

In this work, optimum tube length for turbulent flow which allows highest heat transfer per cross-sectional area of the tube with smooth wall is determined. This problem has not been investigated according to the best knowledge of the author.

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Nomenclature

A	cross-sectional area
c_p	specific heat
d	diameter
h	heat transfer coefficient
k	thermal conductivity
L	length of the duct
L^*	dimensionless length, Eq. (19)
Nu	Nusselt number, Eq. (27)
Pr	Prandtl number
$\dot{q}$	heat flux, Eq. (2)
$\dot{q}^*$	dimensionless heat flux, Eq. (9)
$\dot{Q}$	heat flow
p	pressure
Re	Reynolds number, Eq. (15)
Re_p	pressure Reynolds number, Eq. (25)
RMSE	root mean square error
T	temperature
u	velocity
u_p	pressure velocity, Eq. (6)
$\dot{V}$	volume flow rate
x	axial coordinate
z	entrance number, Eq. (28)

Greek symbols

Δp	pressure drop
ΔT	temperature difference, Eq. (5)
ε	porosity
θ	dimensionless temperature, Eq. (7)
λ	pressure loss coefficient
ν	kinematic viscosity
ρ	density

Superscripts and subscripts

*	dimensionless
1	for $\varepsilon = 1$
ε	for $\varepsilon \neq 1$
e	exit
f	frictional
i	inlet, incremental
l	local
m	mean
o	optimum
p	pressure
w	wall

2. Derivation of the equations

Heat transfer in a tube is formulated as

$$\dot{Q} = \rho c_p \dot{V}(T_i - T_e) \tag{1}$$

where $\rho, c_p, \dot{V}, T_i$ and T_e are density, specific heat, volume flow rate, mean inlet and mean exit temperatures of the fluid, respectively. Heat transfer per cross-sectional area is given with the following equation:

$$\dot{q} = \frac{\dot{Q}}{A} = \rho c_p u_m (T_i - T_e) \tag{2}$$

where A and u_m are cross-sectional area and mean fluid velocity, respectively. They are defined as

$$A = \frac{\pi}{4} d^2 \tag{3}$$

$$u_m = \frac{\dot{V}}{A} \tag{4}$$

Using the definitions

$$\Delta T = T_i - T_w \tag{5}$$

$$u_p = \sqrt{2\Delta p/\rho} \tag{6}$$

$$\theta = \frac{T_e - T_w}{\Delta T} \tag{7}$$

$$u^* = \frac{u_m}{u_p} \tag{8}$$

$$q^* = \frac{\dot{q}}{\rho c_p u_p \Delta T} \tag{9}$$

one obtains from Eq. (2)

$$q^* = u^*(1 - \theta) \tag{10}$$

Here, T_w and Δp are constant wall temperature and total pressure loss in the tube, respectively. Total pressure loss consists of frictional pressure loss Δp_f for developed flow, incremental pressure loss Δp_i and local pressure loss Δp_l. Local pressure loss can be calculated by

$$\Delta p_l = \lambda_l \frac{\rho u_m^2}{2} \tag{11}$$

where λ_l is determined from the porosity ε in heat exchangers [11]:

$$\lambda_l = \frac{(3 - \varepsilon)(1 - \varepsilon)^2}{2 - \varepsilon} \tag{12}$$

ε can be envisaged as the ratio of fluid velocities before entering the tube to that in the tube. Frictional pressure loss at developed flow conditions can be calculated from

$$\Delta p_f = \lambda_f \frac{L}{d} \frac{\rho u_m^2}{2} \tag{13}$$

where L is the length of the tube. Frictional pressure loss coefficient λ_f should be calculated with the equation of Prandtl [12],

$$\frac{1}{\lambda_f} = 2.0 \log\left(Re\sqrt{\lambda_f}\right) - 0.80 \tag{14}$$

where Reynolds number Re is defined as

$$Re = \frac{u_m d}{\nu} \tag{15}$$