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Role of 'Bejan's heatlines' in heat flow visualization and optimal thermal mixing for differentially heated square enclosures

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Abstract

Heat flow patterns in the presence of natural convection have been analyzed with Bejan's heatlines concept. Momentum and energy transfer are characterized by streamfunctions and heatfunctions, respectively such that streamfunctions and heatfunctions satisfy the dimensionless forms of momentum and energy balance equations, respectively. Finite element method has been used to solve the velocity and thermal fields and the method has also been found robust to obtain the streamfunction and heatfunction accurately. The unique solution of heatfunctions for situations in differential heating is a strong function of Dirichlet boundary condition which has been obtained from average Nusselt numbers for hot or cold regimes. The physical significance of heatlines have been demonstrated for a comprehensive understanding of energy distribution and optimal thermal management via analyzing three cases. Case 1 involves the uniform and non-uniform heating of bottom wall with cooled side walls. The studies illustrate that the heat flow primarily occurs from the central regime of the bottom wall to a very small regime of the top portion of side walls. A large portion of central regime of cold side walls do not receive significant amount of heat. In order to maximize the thermal energy distribution, the distributed heating at the middle portions of the bottom and side walls have been considered in case 2 and heatlines clearly depict the distributions of heat from the hot walls to the large regimes of the cold wall. Further case 3 illustrates the enhanced heat flows in presence of heated bottom and left side walls. Heatline is found as an effective numerical tool to visualize energy distribution in order to establish a suitable heating strategy.

Keywords: Heatlines; Heatfunctions; Streamlines; Streamfunctions; Natural convection; Square cavity; Uniform and non-uniform heating

1. Introduction

Natural convection in enclosed cavities has received significant attention due to many engineering applications [1–3]. Steady natural convection within a differentially heated square enclosure has a major role in food preservation at an optimal temperature. Analysis of food sterilization in various cavities has also been studied by earlier investigators [4,5].

Investigations of natural convection in a square enclosure has been carried out for past two decades by several investigators (Patterson and Imberger [6], Nicolette et al. [7], Hall et al. [8], Hyun and Lee [9], Fusegi et al. [10], Lage and Bejan [11,12], and Xia and Murthy [13]). November and Nansteel [14] and Valencia and Frederick [15] have shown a specific interest to focus on a natural convection within a rectangular enclosure wherein a bottom heating and/or a top cooling are involved. Studies on natural convection in rectangular enclosures heated from below and cooled along a single side or both sides have been carried out by Ganzarolli and Milanez [16]. Kimura and Bejan [17] also studied natural convection in differentially heated corner region. They established that the flow field is relatively insensitive to whether the wall temperature varies continuously or discontinuously through the corner point. Later, the case of heating from one side and cooling from

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Nomenclature

g	acceleration due to gravity, m s^{-2}
k	thermal conductivity, W m^{-1} K^{-1}
L	side of the square cavity, m
N	total number of nodes
p	pressure, Pa
P	dimensionless pressure
Pr	Prandtl number
R	residual of weak form
Ra	Rayleigh number
T	temperature, K
T_h	temperature of hot bottom wall, K
T_c	temperature of cold vertical wall, K
u	x component of velocity
U	x component of dimensionless velocity
v	y component of velocity
V	y component of dimensionless velocity
X	dimensionless distance along x coordinate
x	distance along x coordinate
Y	dimensionless distance along y coordinate
y	distance along y coordinate

Greek symbols

α	thermal diffusivity, m^2 s^{-1}
β	volume expansion coefficient, K^{-1}
γ	penalty parameter
Γ	boundary
θ	dimensionless temperature
ν	kinematic viscosity, m^2 s^{-1}
ρ	density, kg m^{-3}
Φ	basis functions
ψ	streamfunction
Π	heatfunction

Subscripts

i	residual number
k	node number

the top has been analyzed by Aydin et al. [18] who investigated the influence of aspect ratio for air-filled rectangular enclosures. Also, Kirkpatrick and Bohn [19] examined experimentally the case of high Rayleigh number natural convection in a water-filled cubical enclosure heated simultaneously from below and from the side. Corcione [20] studied natural convection in a air-filled rectangular enclosure heated from below and cooled from above for a variety of thermal boundary conditions at the side walls. Numerical results were reported for several values of both width-to-height aspect ratio of the enclosure and Rayleigh number. Recently, Basak et al. [21] and Roy and Basak [22] investigated natural convection within a square enclosure for hot bottom wall and various hot/cold side walls with insulated top wall. However, optimal heating policies for all these situations have not been reported. Current work fills the gap by analyzing the 'Bejan's heatlines' for visualizing energy flow due to natural convection in a square enclosure.

The heatline is the best way to visualize the heat transfer in two dimensional convective transport processes. The streamlines are the best tools to visualize the fluid motions in two dimensional incompressible flow. Similarly, the heatlines, which are also heat flux lines, represent the trajectory of heat energy. In general, the heat flux lines are normal to the isotherms for heat transfer due to pure conduction through isotropic media. Energy flow within various regimes especially for convective heat transport processes can be best visualized by heatlines whereas isotherms are unable to give guideline for energy flows. The heatlines are mathematically represented by heatfunctions and the proper dimensionless forms of heatfunctions are closely related to overall Nusselt numbers.

The heatline concept was first introduced by Kimura and Bejan [23] and Bejan [24]. Various applications were further studied by Bello-Ochende [25], Costa [26–29] and Deng and Tang [30]. Bejan [31] also reviewed some earlier works on heatlines and illustrated the use of heatline concept to visualize various physical situations. Till date, the heatline concept has not been used extensively for analyzing convective heat transport processes except for very few applications. Application of heatlines was shown for thermomagnetic convection in electroconductive melts [32,33]. The heatline concept was further used for unsteady heat transfer assuming that steady state version of energy balance equation is satisfied at a given instant [34]. Application of heatlines have also been carried out for investigations in polar coordinates [35–39]. The heatline concept has also been applied for analyzing heat transfer involving forced convection [40,41] and turbulent flows [42].

The aim of this article is to analyze energy flows due to natural convection in a square enclosure with hot bottom wall and cold side walls in presence of insulated top walls. The main objective of this fundamental study is to examine thermal mixing near the central core of the cavity especially for food processing applications. Further, the influence of distributed heat source will be investigated to enhance thermal mixing via the trajectory of heat flow using 'Bejan's heatlines' concept. The thermal mixing and heat flow will also be investigated for two adjacent hot walls with a cold side wall in presence of insulated top wall. In the current study, we have used Galerkin finite element method with penalty parameter to solve the non-linear coupled partial differential equations of flow and temperature fields. The Galerkin method is further employed to solve the Poisson equation for streamfunctions and heatfunctions. It may