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Chaotic modelling and control of combustion instabilities due to vaporization

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ABSTRACT

In this paper, a state of the art LES algorithm is employed to validate an evaporation model to be employed in predictive modelling regarding combustion instabilities. Good agreement between the numerical predictions and experimental data is achieved. Additionally, transient sub-critical droplet evaporation is investigated numerically. In particular, a numerical method is proposed to capture the extremely important pressure–velocity–density coupling. Moreover, a discrete dynamic model accounting for both combustion and vaporization processes is developed. In terms of different bifurcation parameters relevant to either combustion or evaporation, various bifurcation diagrams are presented. As part of the nonlinear characterization, the governing process Lyapunov exponent is calculated and employed to analyze the stability of the particular dynamic system. The study has shown conclusively that the evaporation process has a significant impact on the intensity and nonlinear behaviour of the system of interest, vis-à-vis a model accounting for only the gaseous combustion process. Furthermore, a particular nonlinear control methodology is adopted to control the chaotic behaviour displayed by the particular aperiodic motions observed. This algorithm is intended to be implemented for control of combustion instability numerically and experimentally to provide a rational basis for some of the control methodologies employed in the literature.

1. Introduction

The problem of combustion instability can occur in many different ways with liquid–fuelled gas turbine combustors. The candidate mechanisms for this challenging phenomena result from the interaction between the combustion chamber acoustics and one or more processes which are related to liquid injection, primary atomization, secondary atomization, chemical kinetics, evaporation and liquid heating and mixing [15,54,26]. Particularly, in the past, the vaporization process, as one of the key factors driving combustion instability, has been investigated by large number researchers [47,49,12]. Compared with the other processes associated with combustion chamber, vaporization, in general, is the slowest, and hence may be the rate-controlling process, as pointed out by Sirignano et al. [54]. Recently, the effect of the oscillating gas pressure and velocity on the evaporation rate of droplets was examined by Tong and Sirignano [49] and it was revealed that, under certain circumstances, the vaporization-rate response function can be sufficient to induce combustion instability. More recently, using a detailed calculation of a single droplet surrounded by the oscillating field, Duvvur et al. [12] concluded that for certain frequency ranges and initial droplet sizes, instabilities can arise in an evaporation-rate-controlled chamber owing to the vaporization process. Therefore, the method involving modulations of the evaporation process for liquid fuels has been extensively employed to control combustion instability [38,21,10,33].

The general criterion for wave growth or decay is that the amplitude of wave will increase provided that sufficient energy or mass from combustion and/or evaporation is added in phase with acoustics existing in the combustion chamber, whereas it will reduce if the addition is out of phase [27,15,44,9]. Rayleigh [27] initially investigated the mechanism by which the heat release energy is transferred to the system acoustic modes. A mathematical expression for the Rayleigh criterion was derived by Culick [7,8]. In the past, this has been extensively used to examine the stability of the system of interest numerically and experimentally [34,18,36]. In terms of the vaporization process, a general Rayleigh criterion was evaluated by Sirignano and co-workers [49,12]. The growth and decay of the wave can be determined by the net in-phase or out-of-phase mass addition [44].

In addition, the various nonlinear behaviours observed in combustion instability have been investigated experimentally, analytically and computationally by a number of authors. Culick [5,6] put forward a second-order nonlinear dynamic model using the Galerkin method, which was extended to third-order by Yang et al. [51,52]. This dynamic model has been widely utilized to analyze the nonlinear behaviour inherent in combustion instability for a combustion chamber. The triggering conditions for the different modes in the chamber were given by Yang et al. [51–53]. The bifur-

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Nomenclature

A	coefficients in discretization equations
$\bar{a}$	average speed of sound for the mixture
B	mass transfer number
D	binary diffusion coefficient
E	oscillation energy
E^*	modified oscillation energy
$e_{o,g}$	stagnation internal energy of gaseous species
$e_{o,d}$	stagnation internal energy of droplets
F^*	mass flux at cell surface in discretization equations
$\mathbf{F}_d$	inter-force between droplets and gases
G	Green function
$\hat{h}_{fg}$	latent heat of evaporation
k	thermal conductivity; constant in Section 4.4
k_n	waver number
L	Lyapunov exponent
C_p	specific heat at constant pressure
C_v	specific heat at constant volume
$\overline{C}_v$	specific heats of gas/droplets mixture at constant volume
$\overline{C}_p$	specific heats of gas/droplets mixture at constant pressure
m	mass fraction
$\dot{m}''$	mass flux
r	radial coordinate
R	radius
R_u	universal gas constant
$\overline{R}$	mass average gas constant for the gas/droplets mixture
Re	Reynolds number
p	pressure
$\dot{Q}$	heat released by reactions
S	cell surface area
Sc	Schmidt number
Sh	Sherwood number
t	time
T	temperature
$\mathbf{u}$	velocity vector
v_r	radial velocity
V	modified velocity
W_d	rate of conversion of droplets to gas
x	dimensionless radial coordinate

Greek symbols

α	thermal diffusivity
γ	heat capacity ratio
$\bar{\gamma}$	ratio of heat heats for the gas/droplets mixture
λ	bifurcation parameter
η	amplitude function
$\boldsymbol{\psi}_n$	mode shape
ρ	density
κ	ratio of droplets to the mass of gas in a unit volume of chamber
μ	dynamic viscosity; bifurcation parameter in Section 4
ν	kinematic viscosity
τ	dimensionless time
θ	dimensionless temperature
ξ	dimensionless radius
$\dot{\xi}(\tau)$	$=\mathrm{d}\xi(\tau)/\mathrm{d}\tau$

Subscripts

0	at initial time
1	fuel species
2	air
d	droplet
l	liquid phase
g	gaseous phase
s	surface of droplet
sat	saturation
∞	far away from droplet surface

Superscripts

$'$	correction part in Section 2; fluctuating part in Section 4
p	period in Section 4.4

cation phenomena and the transitional process were reported by Sterling [46]. The limit cycle occurring in solid rocket motors was discussed using a numerical method by Levine and Baum [24]. The bifurcation occurring in a dump combustor was confirmed by experiments [25]. More recently, to fully understand the underlying mechanisms relevant to combustion instability, nonlinear analyses have been employed in a more comprehensive manner. Bifurcation of the steady combustion regimes induced by combustion in a dump combustor was pointed out by Dubinkin et al. [11]. Huang et al. [17] argued that the heat transfer coefficient between the wall and the burned gas is an important bifurcation parameter for the combustion instability. Ananthkrishnan et al. [2] provided the dependence of the inter-modal energy transfer on the relevant parameters and reported the existence of a Hopf bifurcation related to thermo-acoustic instability. The bifurcation behaviour of a flame in a combustion chamber was recognized using a large eddy simulation (LES) method by Huang and Yang [18]. The fractal dimension of the corresponding attractors associated with the acoustic modes participating in the pressure oscillations in a combustion chamber was outlined by Sterling [46]. Lei and Turan [23] identified the nonlinear/chaotic behaviour inherent in thermo-acoustic instability using dynamic models based on reasonable considerations. The hysteresis inherent to combustion instability was revealed in addition to other interesting phenomena.

Since the evaporation process is one of the key mechanisms driving combustion instability in gas turbines and rockets in particular, as pointed out by Sirignano and co-workers [1,12,44,54], this crucial process is investigated in a progress update manner in the paper. Furthermore, this study is primarily focused on studying the evaporation process relevant to combustion instability from three complementary viewpoints in an effort to contribute to an overall instability model driven primarily by evaporation in gas turbine combustors. The study starts with transient evaporation model of a single fuel droplet by means of a detailed numerical method. It should be noted that, the pressure–velocity–density coupling is taken into consideration which hitherto has been neglected. In addition, a state of the art LES algorithm is employed to validate the evaporation model for use in predictive studies regarding combustion instabilities. As part of the effort, a discrete dynamic model is developed that accounts for both the combustion and the evaporation processes as distinct from models employing just the typical premixed combustion description. A detailed analysis of the relevant nonlinear behaviour including instability is carried out. These are of particular significance to delineate the comprehensive mechanisms for the combustion instability since the bifurcation behaviour and the multiplicity of solutions for a given nonlinear system are seldom concerned in the currently employed experimental and numerical studies (CFD) in terms of