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Separation in an inclined porous thermogravitational cell

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ABSTRACT

This paper reports a theoretical and numerical study of species separation in a binary liquid mixture saturating a shallow porous layer heated from below or from above and inclined with respect to the vertical axis. It is shown that the separation can be increased using this configuration and the stability of the unicellular flow obtained in this case is investigated. The critical Rayleigh number obtained is much higher than the one leading to the maximum separation. Experiments performed with a solution of $CuSO_4$ give results which are almost in good agreement with the analytical and the numerical results.

1. Introduction

Thermogravitational diffusion is the combination of two phenomena: convection and thermodiffusion. The coupling of these two phenomena leads to species separation. In 1938, Clusius and Dickel [1] successfully carried out the separation of gas mixtures in a vertical cavity heated from the side (thermogravitational column, TGC). During the following years, two fundamental theoretical and experimental works on species separation in binary mixtures by thermogravitation were published. Furry et al. [2] (FJO theory) developed the theory of thermodiffusion to interpret the experimental processes of isotope separation. Subsequently, many works appeared, aimed at justifying the assumptions or extending the results of the theory of FJO to the case of binary liquids [3]. Other works were related to the improvement of the experimental devices to increase separation. Lorenz and Emery [4] proposed the introduction of a porous medium into the cavity. Platten et al. [5] used an inclined cavity, heated from the top, to increase separation. Elhajjar et al. [6] used a horizontal cavity heated from above with temperature gradients imposed on the horizontal walls to improve the separation process with the use of two control parameters. Bennacer et al. [7] suggested splitting the column into three sub-domains in order to increase the separation. The theory developed by Furry et al. [2] for the separation of isotopes in vertical columns differentially heated on the two vertical walls, showed that there is a maximum of separation for an optimal value of the cell thickness. However, in practice, this thickness is of the order of a fraction of mm which significantly reduces the amount of separated species. If we use cells of larger thickness, the separation decreases since the velocity of flow becomes very high in comparison with the velocity leading to the maximum separation. One way to decrease the velocity and to increase the separation is to tilt the cell by a given angle from the vertical. In this case, the horizontal component of the temperature gradient decreases, which decreases the buoyancy force and the velocity of the flow. The tilted cell has already been used by De Groot [3] in the case of bulk fluid but for different forms of the cells.

In this work, an analytical and numerical study of the separation in a porous cell filled with a binary mixture is performed for different inclinations from the vertical axis. Two configurations are considered: cell heated from the top or from the bottom. A linear stability analysis of the unicellular flow leading to separation is presented. Some experiments are also performed in order to corroborate the theoretical and the numerical results.

2. Mathematical formulation

In binary fluid mixtures subjected to temperature gradients, the thermodiffusion effect induces a mass fraction gradient. In the expression of the mass flux, $\vec{J}$, of one of the components, in addition to the usual isothermal contribution given by the Fick law,

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Nomenclature

A	aspect ratio of the cavity
a^*	thermal diffusivity of the mixture $a^* = \lambda^*/(\rho c)_f$
C	mass fraction of the denser component of the mixture
C_i	initial mass fraction of the denser component of the mixture
D^*	mass diffusion coefficient ($m^2\,s^{-1}$)
D^*_T	thermodiffusion coefficient ($m^2\,s^{-1}\,K^{-1}$)
e	length of the cavity along the axis $\vec{e}_x$ (m)
k	wave number
K	permeability of the porous medium (m^2)
L	length of the cavity along the axis $\vec{e}_z$ (m)
Le	Lewis number $Le = a^*/D^*$
m	gradient of mass fraction along the z-axis
P	pressure of fluid (Pa)
Ra	thermal filtration Rayleigh number $Ra = [Keg\beta_T\Delta T(\rho c)_f]/(\lambda^*\nu)$
Ra_c	critical Rayleigh number associated with transition from equilibrium solution to unicellular flow
S	separation
T	temperature (K)
t	non-dimensional time
$\vec{V}$	velocity of the flow ($m\,s^{-1}$)
u, v	velocity components ($m\,s^{-1}$)

Greek symbols

β_T	thermal expansion coefficient (K^{-1})
β_C	solutal expansion coefficient
ε^*	porosity of porous medium
ε	normalized porosity
ψ	separation ratio $\psi = -(\beta_C/\beta_T)(D^*_T/D^*)C_0(1-C_0)$
σ	temporal amplification of perturbation
λ^*	effective thermal conductivity of the porous medium-mixture system ($W\,m^{-1}\,K^{-1}$)
$(\rho c)_f$	volumetric heat capacity of the mixture ($J\,m^{-3}\,K^{-1}$)
$(\rho c)^*$	effective volumetric heat capacity of porous medium-mixture system ($J\,m^{-3}\,K^{-1}$)
ν	kinematics viscosity of mixture ($m^2\,s^{-1}$)

Superscripts and subscripts

nd	non-dimensional
0	initial value
*	equivalent thermophysical properties of the porous medium

there is an additional contribution proportional to the temperature gradient

$$\vec{J} = -\rho D\vec{\nabla}C - \rho C(1-C)D_T\vec{\nabla}T$$

where D is the mass diffusion coefficient, D_T the thermodiffusion coefficient, ρ the density, and C the mass fraction of the denser component.

We consider a rectangular cell of aspect ratio $A = L/e$ where L is the length of the cell along the axis $\vec{e}_z$ and e is its width along the axis $\vec{e}_x$. The cavity is filled with a porous medium saturated with viscous binary liquid with density ρ and dynamic viscosity μ. The Soret effect is taken into account.

The cavity is inclined at an angle α from the vertical. The gravity vector is $\vec{g} = -g\vec{k}$ where $\vec{k} = -\sin(\alpha)\vec{e}_x + \cos(\alpha)\vec{e}_z$ (Fig. 1). The impermeable walls ($x = 0, x = e$) are kept at different and constant temperatures T_1 for $x = 0$ and T_2 for $x = e$, with $T_1 < T_2$. The walls ($z = 0, z = L$) are impermeable and insulated. All the boundaries are assumed rigid.

We assume that Darcy's law and the Boussinesq approximation are valid, and that the fluid and the solid phases are in local thermal equilibrium. Dufour effect is neglected as has been done by many authors (see for example [8]) because of its minor influence in liquid mixtures. We set all the properties of the binary fluid constant except the density ρ in the buoyancy term, which depends linearly on the local temperature and the mass fraction:

$$\rho = \rho_0[1 - \beta_T(T - T_0) - \beta_C(C - C_0)] \tag{1}$$

Where β_T and β_C are the coefficients of thermal and solutal expansion, ρ_0 the fluid mixture reference density at temperature T_0 and mass fraction C_0.

Subject to these constraints, the governing conservation equations for mass, momentum, energy and chemical species are:

$$\begin{cases} \vec{\nabla}.\vec{\mathbf{V}} = 0 \\ \vec{\nabla}P = \rho\vec{g} - \frac{\mu}{K}\vec{\mathbf{V}} \\ (\rho c)^*\frac{\partial T}{\partial t} + (\rho c)_f\vec{\mathbf{V}}.\vec{\nabla}T = \vec{\nabla}.(\lambda^*\vec{\nabla}T) \\ \varepsilon^*\frac{\partial C}{\partial t} + \vec{\mathbf{V}}.\vec{\nabla}C = \vec{\nabla}.\left[D^*\vec{\nabla}C + C(1-C)D^*_T\vec{\nabla}T\right] \end{cases} \tag{2}$$

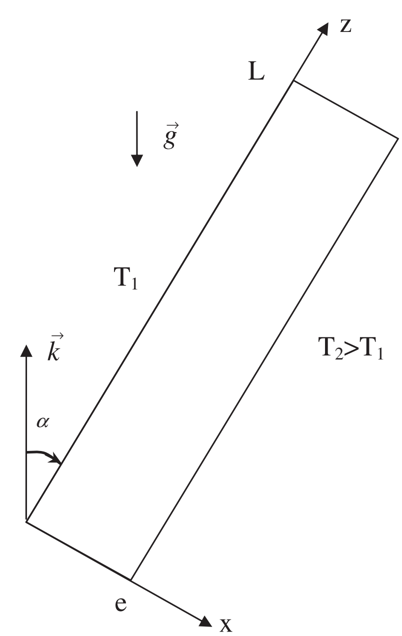

Fig. 1. Geometrical configuration of the inclined cell.

We assume that there is little variation in the term $C(1-C)$ of the equation of conservation of species, so we can replace it by $C_0(1-C_0)$, where C_0 is the initial mass fraction. The variables are non-dimensionalized with: e for the length, a^*/e for the velocity, $(re^2)/a^*$ for the time, $(\mu a^*)/K$ for the pressure (with $r = (\rho c)^*/(\rho c)_f$, where $(\rho c)^*$ is the effective volumetric heat capacity of the porous medium, and a^* the effective thermal diffusivity of the porous medium), $\Delta T = T_2 - T_1$ for temperature ($T_{nd} = (T - T_1)/\Delta T$) and $\Delta C = \Delta TC_0(1-C_0)D^*_T/D^*$ for the mass fraction ($C_{nd} = (C - C_0)/\Delta C$), where D^*_T, D^* are the thermodiffusion and the mass diffusion coefficients of the denser component of mass fraction C.

Thus the dimensionless governing conservation equations for mass, momentum, energy and chemical species are: