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First and second law analysis of fully developed gaseous slip flow in trapezoidal silicon microchannels considering viscous dissipation effect

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ABSTRACT

Fully developed gaseous slip flow in trapezoidal silicon microchannels is studied. Friction factor, Nusselt number and entropy generation in the microchannel is obtained, effect of rarefaction, aspect ratio and viscous dissipation is explored and, the range of Brinkman number in which viscous dissipation effect is important and cannot be neglected is specified. The continuum approach with the velocity slip and temperature jump condition at the solid walls is applied to develop the mathematical model of problem in the trapezoidal microchannel. Transformation of trapezoidal geometry to a square provided ease of application of finite difference method in solution of the mathematical model. The effect of viscous dissipation is quantified by Brinkman number. The calculated Brinkman number for common engineering applications even with limiting operational and geometric conditions is found less than 0.005. It is observed that viscous effect for applications with Brinkman number less than 0.005 can be neglected. The region in which viscous dissipation effect cannot be neglected is specified as $Br > 0.005$. Decreasing effect of rarefaction and increasing effect of Brinkman number on irreversibility due to all sources, excluded axial conduction, is established. The dominant source of irreversibility in total irreversibility is specified as a function of Brinkman number.

1. Introduction

Studies on gaseous flow in microchannels are motivated by rapidly growing micro-fluidic applications in widely extending engineering systems. The practically used microchannels with non-circular cross section are those with rectangular, trapezoidal and hexagonal (double-trapezoidal) cross section shapes. The shape of cross section is a result of microchannel production technology. Morini et al. [1] describe the chemical etching that is the most commonly used technique for building microchannels on silicon wafers. The present study on trapezoidal microchannels is motivated by the fact that the microchannels etched in ⟨1 0 0⟩ silicon using a KOH solution have trapezoidal cross section. Friction factor and heat transfer (Nusselt number) analysis are two ultimate purposes of microchannel studies. A summary of such studies on gaseous flow in trapezoidal microchannels is given below.

Morini et al. [1] studied the rarefaction and cross-section geometry effects on the friction factor of an incompressible gaseous flow in silicon microchannels having a rectangular, trapezoidal and double-trapezoidal cross section. It is found that for the trapezoidal and double-trapezoidal microchannels, the influence of the aspect ratio on the friction factor is strong only if the aspect ratio is less than 0.5. Experimental works of Ding et al. [2], Harley et al. [3] and Araki et al. [4] on gaseous flow in trapezoidal microchannels showed that friction factor is lower than that predicted by conventional theory. Cao et al. [5] studied fully developed laminar slip flow and heat transfer in trapezoidal microchannels with uniform wall heat flux boundary condition. The authors claim that the aspect ratio and base angle have significant effect on flow and heat transfer in trapezoidal micro-channel. The same result is found by Niazmand et al. [6] who examined gas rarefaction effect in simultaneously developing 3D laminar flow in trapezoidal microchannel with constant wall temperature. The study showed that fully developed momentum and heat transfer rates are significantly affected by the degree of rarefaction, aspect ratio and side angle. Both the friction and heat coefficients monotonically decrease with increasing *Kn* and aspect ratio, and decreasing side angles. The friction coefficient shows the strongest dependence on rarefaction effects and weakest dependence on the channel side angle. The heat transfer coefficient shows less sensitivity to rarefaction effects with increasing channel aspect ratio.

Two points are notable regarding the last two works [5,6]. First, the authors assume the base angle as a parameter and give variation of friction factor and Nusselt number with base angle. This assumption is not realistic for silicon microchannels since the base angle is imposed by microchannel production technology and has a fixed value [1]. Second, the problem is solved with uniform heat

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Nomenclature

a	nondimensional long base of microchannel
a_i	long base of microchannel (m)
A_c	cross sectional area (m^2)
$A_{c,\xi\eta}$	nondimensional cross sectional area
b	nondimensional short base of microchannel
b_i	short base of microchannel(m)
Br	Brinkman number (dimensionless)
c_P	Specific heat (J/kgK)
D_h	hydraulic diameter ($4A_c/\Gamma_i$) (m)
f	Fanning friction factor (dimensionless)
F_t	thermal accommodation coefficient (dimensionless)
F_v	tangential momentum accommodation coefficient (dimensionless)
h	convective heat transfer coefficient (W/m^{2}K)
h	nondimensional height of microchannel
h_i	height of microchannel (m)
k	thermal conductivity (W/mK)
Kn	Knudsen number (dimensionless)
ℓ	perimeter of microchannel (m)
ℓ_h	heated perimeter of microchannel (sum of the lengths of top and the two side walls) (m)
L_h	nondimensional heated perimeter of microchannel
n_i	normal to the inside boundaries
N_C	local entropy generation number due to fluid axial conduction (dimensionless)
$\overline{N}_C$	average entropy generation number due to fluid axial conduction (dimensionless)
N_F	local entropy generation number due to fluid friction (dimensionless)
$\overline{N}_F$	average entropy generation number due to fluid friction (dimensionless)
N_S	local entropy generation number (dimensionless)
$\overline{N}_S$	average entropy generation number (dimensionless)
N_V	local entropy generation number due to viscous heating (dimensionless)
$\overline{N}_V$	average entropy generation number due to viscous heating (dimensionless)
$N_{\xi,\eta}$	local entropy generation number due to heat transfer in the ξ and η directions (dimensionless)
$\overline{N}_{\xi,\eta}$	average entropy generation number due to heat transfer in the ξ and η directions (dimensionless)
Nu	Nusselt number (dimensionless)
p	fluid pressure (*Pa*)
P	normalized pressure gradient (dimensionless)
Pe	Peclet number (dimensionless)
Pr	Prandtl number (dimensionless)
q	Heat flux (W/m^2)
R	specific heat ratio (dimensionless)
Re	Reynolds number (dimensionless)
$\dot{S}'''$	local rate of entropy generation, (W/m^{3}K)
$\dot{S}_C'''$	characteristic entropy generation rate (W/m^{3}K)
T	nondimensional temperature
u	nondimensional fluid velocity
v_X, v_Y, v_Z	velocity components on X, Y, Z directions (m/s)
w	fluid velocity (m/s)
x, y, z	transformed coordinates
X, Y, Z	Cartesian coordinates

Greek symbols

α	thermal diffusivity (m^2/s)
α	apex angle (degree)
β	nondimensional variable defined by Eq. (28)
β_t	nondimensional variable defined by Eq. (24)
β_v	nondimensional variable defined by Eq. (23)
$\Delta\theta$	temperature difference (K)
ϕ	viscous dissipation function (s^{-2})
ϕ^*	nondimensional viscous dissipation function
Φ	constant for fully developed flow defined by Eq. (11) (dimensionless)
γ	aspect ratio (dimensionless)
Ω	nondimensional temperature difference
Γ_i	Inside periphery of microchannel (m)
Γ	nondimensional inside periphery of microchannel
λ_{mfp}	molecular mean free path (m)
μ	dynamic viscosity (Pa s)
ν	kinematic viscosity (m^2/s)
θ	temperature (K)
ρ	density (kg/m^3)
ξ, η, ζ	nondimensional coordinates

Subscripts

ave	average
b	bulk property
m	mean value
s	fluid property near the wall
w	wall value
0	inlet property

flux or temperature imposed on four walls of microchannel. Such boundary condition is not realistic for silicon trapezoidal microchannels because the production technology requires covering the long wall with Pyrex glass to bond the silicon, the condition on which cannot be similar to the three other walls. Adiabatic condition is much realistic for this wall [7,8]. A motivation of the present work is to provide knowledge on friction factor and Nusselt number for fully developed gaseous flow in trapezoidal microchannels with heated three walls and adiabatic long base wall.

The effect of viscous dissipation is mostly neglected for gaseous flows in microchannels. From conventional theory it is well known that viscous dissipation effect becomes important if; flow velocity or fluid viscosity or both are high. Flow velocity or fluid viscosity may not be high for gaseous microchannel flows, however, viscous dissipation effect may still be significant for such flows. Because in addition to high velocity or high fluid viscosity, viscous dissipation is also significant if a small wall-to-fluid temperature difference exists, which is the case for microchannel flows [9]. Conversion of kinetic energy to thermal energy as a result of viscous dissipation increases the temperature of flowing fluid. This in turn decreases/increases the temperature difference between the wall and flowing fluid when the fluid is heated/cooled, an immediate result of which is decrease/increase in heat transfer which is an ultimate purpose for microchannel applications. Such effect guides one to take into account viscous dissipation where it is significant. In addition to providing knowledge on friction factor and Nusselt number for fully developed gaseous flow in trapezoidal microchannels, another major purpose of the present paper is to; determine the ranges where viscous dissipation is significant and, investigate its effect on heat transfer where it is significant. Studies dealing with viscous dissipation effect on gaseous flow and heat transfer in microchannels are summarized below.

Maynes and Webb [10] studied the influence of viscous dissipation on thermally fully-developed, electro-osmotically generated flow in a parallel plate microchannel and circular microtube. It is found that for practical electro-osmotic flow/heat transfer applica-