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Transition to a periodic flow induced by a thin fin on the sidewall of a differentially heated cavity

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ABSTRACT

The transition to a periodic flow induced by a thin fin on the sidewall of a differentially heated cavity is numerically investigated. The numerical results are compared with a previously reported experiment. It is demonstrated that the transient flow obtained numerically shows features consistent with the experimental flow. Based on the present numerical results, the temporal development and spatial structures of the thermal flow around the fin are described, and the separation of the thermal flow above the fin is discussed. It is found that the presence of the fin changes the flow regime and results in the transition of the thermal flow to a periodic flow. The present numerical results also indicate that the unstable temperature configuration above the fin results in intermittent plumes at the leeward side of the fin, which in turn induce strong oscillations of the downstream boundary layer flow. It is demonstrated that the oscillations of the boundary layer flow significantly enhance the heat transfer through the finned sidewall (by up to 23%).

1. Introduction

Natural convection in a differentially heated cavity is one of the classical problems of fluid mechanics and heat transfer, and has wide industrial application. Consequently it has been extensively studied over the past decades. One of the earliest studies of this problem was reported by Bachelor [1], who demonstrated that for sufficiently small Rayleigh numbers, the mode of heat transfer is primarily dominated by conduction. Subsequent investigations (e.g. [2,3]) have focused on steady natural convection flows in the differentially heated cavity.

However, natural convection in industrial systems is usually unsteady. Accordingly, the transition of the natural convection flow in the cavity following sudden heating has been given considerable attention over the last two decades. Based on a scaling analysis, Patterson and Imberger [4] pointed out that the base flow during the transition mainly involves a vertical boundary layer flow, a horizontal intrusion and the flow in the core. In the case of low Rayleigh numbers (smaller than a critical value), the transition is characterized by the following processes: (a) the transition of the vertical boundary layer from unsteady one-dimensional to steady two-dimensional, which is marked by an overshoot of the temperature signal and subsequent travelling waves induced by the leading edge effect (LEE) [5–7]; (b) the formation of horizontal intrusions due to the presence of the horizontal walls; (c) the arrival of the horizontal intrusion from the opposite sidewall which triggers the second group of travelling waves in the vertical boundary layer [8,9]; and (d) the approach to a steady state and the stratification of the core flow [8,10]. In the case with Rayleigh numbers larger than the critical value, it has been reported that the natural convection flow in the cavity approaches a periodic flow [11–13]. If the Rayleigh number is sufficiently large, the natural convection flow in the cavity even becomes fully turbulent, as reported in [14,15].

The flow in different flow regimes determines heat transfer through the cavity, and thus it is possible to either enhance or depress heat transfer by manipulating the transition of the flow. One of the simplest techniques for enhancing or depressing heat transfer through a differentially heated cavity is to place a horizontal fin on the heated or cooled sidewall, which has been extensively reported in the literature. In most of the previous studies [16–18], the thickness of the fin is considered to be negligible or small in comparison with the fin length (the so-called thin fin), and the effect of the fin length on the natural convection flow in the cavity is considered. If the length of a fin is sufficiently large, secondary circulations arise at both the upper and lower corners of the fin [19]. It is reported that the heat transfer through the finned sidewall is reduced as the fin length increases due to the depression of the natural convection flow adjacent to the finned sidewall [19–21]. However, Ooshuizen and Paul [22] revealed that the secondary circulations resulting from the presence of a large thin fin on one wall of the cavity enhance convective flows adjacent to the opposite sidewall and thus enhance the heat transfer through the opposite sidewall.

It is noted that steady laminar natural convection flows at low Rayleigh numbers induced by a fin are the focus of the early studies

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Nomenclature

A	Aspect ratio, H/L	T_0	Initial temperature (K)
g	Acceleration due to gravity (m/s^2)	T_c,T_h	Temperatures of the cold and hot sidewalls (K)
H,L	Height and length of the cavity (m)	T_{max},T_{min}	Maximum and minimum temperatures of the fluid layer above the fin (K)
k	Thermal conductivity (W/m/K)	ΔT	Initial temperature difference between the working fluid and sidewall (K)
Nu_{fin}	Spatially averaged Nusselt number of the hot sidewall with a fin	u, v	Velocity components in the x and y directions (m/s)
Nu_{nofin}	Spatially averaged Nusselt number of the hot sidewall without a fin	x, y	Horizontal and vertical coordinates (m)
p	Pressure (N/m^2)	Δy	Thickness of the unstable fluid layer (with an adverse temperature gradient) above the fin
Pr	Prandtl number, ν/κ	β	Coefficient of thermal expansion (1/K)
Ra	Rayleigh number, $g\beta\Delta TH^3/\nu\kappa$	ε	Enhancement factor of heat transfer
Ra_{loc}	Local Rayleigh number, $g\beta(T_{max}-T_{min})\Delta y^3/\nu\kappa$	κ	Thermal diffusivity (m^2/s)
t	Time (s)	ν	Kinematic viscosity (m^2/s)
Δt	Time-step (s)	ρ	Density (kg/m^3)
T	Temperature (K)		

[18]. Since the Rayleigh number plays an important role in the natural convection flow in the cavity [3,11,14], it is of fundamental and practical significance to examine the effect of the fin on the flow and heat transfer over an extended range of Rayleigh numbers. In particular, it is necessary to investigate the transition of the natural convection flow to steady state induced by a fin at high Rayleigh numbers. Accordingly, a shadowgraph observation of the transient natural convection flow in a suddenly differentially heated cavity with a small square fin on the heated sidewall was recently performed by Xu et al. [23], who classified the transition of the flow resulting from sudden heating into three distinct stages: an early stage, a transitional stage and a quasi-steady stage. It is found that, in the early stage, the fin blocks the upstream vertical boundary layer flow and forces it to detach from the finned sidewall, and thus a lower intrusion front is formed. The lower intrusion front almost immediately reattaches to the downstream sidewall after it bypasses the fin. A double-layer structure of the vertical boundary layer, similar to that observed without a fin [24], is ultimately formed in the transition to the quasi-steady state. The observed flow features are also consistent with those reported in [25], in which no clear separation around the small square fin in the laminar flow regime is observed.

As indicated in the previous studies [18], the fin length is an important parameter affecting the natural convection flow in the cavity, and thus a further experiment has been performed by the present authors [26] in order to investigate the effect of the fin length on the transient natural convection flow in the cavity. It is found that the transition of the natural convection flow induced by a large thin fin exhibits features distinct from that induced by a small square fin, and flow separation and oscillations of the thermal flow above the thin fin have been observed. Furthermore, oscillations in turn trigger travelling waves in the downstream boundary layer and the potential for transition to a turbulent downstream flow. However, since the experimental observations provide only qualitative information on the transient flow, it is necessary to perform a further quantitative investigation in order to obtain insights into the correlation between the fin and the flow separation and oscillations around the fin. This motivates the present numerical simulation.

In this paper, the experimental set-up reported in [26] is numerically simulated by a two-dimensional cavity. The numerical procedures are described in Section 2; numerical results are compared with the experimental data in Section 3; the oscillations of the thermal flow around the fin are examined in detail in Section 4; and the enhancement of heat transfer is calculated in Section 5. Finally, the conclusions are presented in Section 6.

2. Numerical procedures

The experiment by Xu et al. [26], whose experimental model is sketched in Fig. 1(a), is considered. The walls of the cavity are made of 19.5-mm thick acrylic sheet (Perspex^{TM}) except for the two 1-mm thick copper sidewalls adjacent to the water baths. A 2-mm thick acrylic fin of length 40 mm is attached horizontally at the mid-height of the heated sidewall, as seen in Fig. 1(a). Since the thermal conductivity of acrylic sheet is only about 0.2 Wm^{-1} K^{-1}, much less than that of copper (385 Wm^{-1} K^{-1}) [26], the heat transfer through the acrylic walls and fin is negligibly small in comparison with that through the copper sidewalls. Accordingly, the acrylic walls and fin are considered adiabatic and the two copper sidewalls are regarded as isothermal in the present numerical simulation.

Previous studies [8,11] show that two-dimensional simulations are able to characterize well the flow features of the transient natural convection in the cavity. Furthermore, since the shadowgraph procedure used in [26] presents images which are essentially transverse integrals of the flow, it is able to describe only the two-dimensional structure of the flow. Accordingly, a two-dimensional numerical simulation is performed in this paper by solving the two-dimensional governing equations with the Boussinesq approximation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\beta(T-T_0), \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\kappa\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

In the experiment described by Xu et al. [26] the working fluid (water) is initially motionless and isothermal with a temperature T_0 = 295.55 K, which is thus adopted as the initial condition in the present numerical simulation. Once the experiment starts (at t = 0), the temperature of the sidewall with the fin is suddenly raised by ΔT by the water in the hot water bath and the tempera-