International Journal of Heat and Mass Transfer 52 (2009) 709–715

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Stability of conducting viscous film flowing down an inclined plane with linear temperature variation in the presence of a uniform normal electric field

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ARTICLE INFO

Article history:
Received 1 January 2008
Available online 15 September 2008

Keywords:
Thin film
Finite amplitude stability analysis
Electrodynamic instability
Marangoni instability

ABSTRACT

Weakly nonlinear stability analysis of a thin liquid film falling down a heated inclined plane with linear temperature variation in the presence of a uniform normal electric field has been investigated within the finite amplitude regime. A generalized kinematic equation for the development of free surface is derived by using long wave expansion method. A normal mode approach and the method of multiple scales are used to investigate the linear and weakly nonlinear stability analysis of film flow, respectively. It is found that both Marangoni and electric Weber numbers have destabilizing effect on the film flow. The study reveals that both supercritical stability and subcritical instability are possible for this type of film flow. It is interesting to note that both the Marangoni and electric Weber numbers have qualitatively same influence on the stability characteristics but the effect of Marangoni number is much stronger compare to the electric Weber number. Scrutinizing the effect of Marangoni and electric Weber numbers on the amplitude and speed of waves it is found that, in the supercritical region amplitude and speed of the nonlinear waves increases with the increase in Marangoni and electric Weber numbers, while in the subcritical region the threshold amplitude decreases with the increase in Marangoni and electric Weber numbers. Finally, we obtain that spatially uniform solution is side-band stable in the supercritical region for our considered parameter range.

1. Introduction

The effect of electric field on a thin liquid film produces a class of problems that has attracted much attention of several researchers due to its technological applications. The presence of electric field introduces additional physical effects on the flow dynamics such as body force due to a current in conducting fluids and the Maxwell stress at the free interfaces. In the industrial process electric field has been used to destabilize the liquid film on the plane. Kim et al. [1] studied the interaction of an electrostatic field on the film flowing in an inclined plane. They found that, in the thin film limit, the effect of electric field occurred in the wave evolution as an external pressure distribution, which causes destabilizing effect on the film. It is interesting to note that, they provide a discussion on the application of their results to a proposed electrostatic liquid film space radiator. Nonlinear stability of a perfectly conducting film flowing down an inclined plane in presence of normal electric field was investigated by González and Castellanos [2]. They derived a nonlinear evolution equation with a Hilbert transform type of term within the limit of small Reynolds number and predict the destabilizing effect of the electric field in the finite amplitude regime. Recently, Mukhopadhyay and Dandapat [3] extended the study of González and Castellanos [2] within the regime of large Reynolds number and confirmed the existence of subcritical unstable and supercritical stable zones. They have also determined a critical value of the electric parameters below which the flow remains stable.

Another set of problems concerns the thermocapillary effect on the falling film down an inclined plane. The interfacial stress generated by the surface tension gradient (Marangoni effect) and the associated modes of instability are known as thermocapillary instability. Gousiss and Kelly [4] investigated the effect of thermocapillarity on a liquid film falling down an inclined uniformly heated plane by performing a linear stability analysis based on Orr–Somerfield and linearized energy equation. They found that a heated wall has a destabilizing effect on the free surface but a cooled wall stabilizes the flow. Later Miladinova et al. [5] studied the effect of non-uniform heating of the plane in the finite amplitude regime by long wave expansion method, as a consequence their study is valid only at a small vicinity of the critical Reynolds number. To overcome this limitation Kalliadasis et al. [6] have studied the problem by integral boundary layer method but they have considered the plane to be uniformly heated. This integral boundary layer method has an inherent error as it does not accurately predict the behaviour of the film close to criticality, such as the order of 20% for the critical Reynolds number. Ruyer-quil et al. [7] studied the problem by higher-order weighted residual

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doi:10.1016/j.ijheatmasstransfer.2008.06.043

approach with polynomial expansions for both velocity and temperature field to overcome the limitation about the criticality of the above problem. In spite of some limitation each study has a great importance in its own to accelerate the ongoing research in the respective field. Recently, Mukhopadhyay and Mukhopadhyay [8] investigated the influence of thermocapillarity on the span of supercritical/subcritical regimes and showed that it has a strong effect on the amplitude and speed of the nonlinear waves. The review of the literature confirms that, there is no such study addressing the effect of the above two types of problems simultaneously. In the present study, an attempt is made to consider the combined effect of uniform normal electric field that at infinity on the flow of conducting viscous film on an inclined heated plane with linear temperature variation.

2. Formulation of the problem

Consider a layer of a conducting thin liquid film flows down an inclined heated plane of inclination θ with the horizon under the action of gravity and an uniform electric field that at infinity and perpendicular to the unperturbed interface. The co-ordinate system is chosen such that x-axis along the flow and z-axis normal to the inclined plane. We assume that the electrical permittivities are constant but take different value in different medium. Due to constant permittivity, the fluid is not coupled to the electric field in the bulk (Melcher and Taylor [9]). However, electrical parameters suffer discontinuities at the interfacial region only, so interface experiences the effect of electric field. The governing equations consist of the continuity equation, Navier–Stokes equation for the flow of the liquid layer, energy equation for the temperature field and Laplace equation for the electric field. The governing equations in dimensional form can be written as:

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

$$\rho(\mathbf{v}_t + (\mathbf{v}\cdot\nabla)\mathbf{v}) = -\nabla p + \rho\nu\nabla^2\mathbf{v} + \rho\mathbf{g} \tag{2}$$

$$\partial_t T + (\mathbf{v}\cdot\nabla)T = \kappa\nabla^2 T, \tag{3}$$

$$\nabla^2\Phi = 0 \tag{4}$$

where $\mathbf{v} = (u, 0, v)$ is the velocity vector, $\mathbf{g} = (g\sin\theta, 0, -g\cos\theta)$ is the acceleration due to gravity vector and p, ρ, ν, T denote the pressure, density, kinematic viscosity and absolute temperature, respectively, and $\nabla = (\partial/\partial x, 0, \partial/\partial z)$. Also $\kappa = k_T/(\rho C_p)$ denote the thermal diffusivity, k_T thermal conductivity, C_p the specific heat at constant pressure of the fluid and Φ denotes the electric potential. The pertinent boundary conditions on the inclined plane ($z = 0$) and at the free surface ($z = h(x,t)$) are:

No-slip condition at the plane:

$$\mathbf{v} = 0 \quad \text{at} \quad z = 0, \tag{5}$$

law of temperature variation of the plane:

$$T = T_g + \mathscr{A}x \quad \text{at} \quad z = 0, \tag{6}$$

kinematic boundary condition:

$$\partial_t h + (\mathbf{v}\cdot\nabla)(h - z) = 0 \quad \text{at} \quad z = h, \tag{7}$$

condition that the liquid is grounded perfect conductor:

$$\Phi = 0 \quad \text{at} \quad z = h, \tag{8}$$

continuity of the shear stress:

$$[[\mathbf{n}\cdot\tau\cdot\mathbf{t}]] = \nabla\sigma(T)\cdot\mathbf{t} \quad \text{at} \quad z = h, \tag{9}$$

continuity of the normal stress:

$$[[\mathbf{n}\cdot\tau\cdot\mathbf{n}]] - [[p]] = -\sigma(T)\nabla\cdot\mathbf{n} \quad \text{at} \quad z = h, \tag{10}$$

Newton's law of cooling:

$$k_T\nabla T\cdot\mathbf{n} + k_g(T - T_g) = 0 \quad \text{at} \quad z = h, \tag{11}$$

where T_g denotes the temperature in the gas phase, $\mathscr{A} = (T_H - T_C)/l_0$, where T_H and T_C denote the temperatures at hotter part and the colder part, respectively, along the inclined plane and l_0 the characteristic longitudinal length scale whose order may be considered same as the wave length λ. In this study, we have taken the temperature T is increasing in the stream-wise direction and hence $\mathscr{A}$ is positive. Also $\sigma(T)$ is the surface tension of the liquid, k_g is the heat transfer coefficient between the liquid and air and [[*]] denotes a jump in the quantity as the interface is crossed from the liquid to vacuum region. $\mathbf{n}$ and $\mathbf{t}$ are the normal and tangent vectors pointing outward to the interface, respectively, and the stress tensor $\boldsymbol{\tau}$ is given by

$$\boldsymbol{\tau} = \boldsymbol{\tau}^f + \boldsymbol{\tau}^e$$

where the viscous stress tensor

$$\tau^f_{ij} = \rho\nu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

and the electrical (Maxwell) stress tensor

$$\tau^e_{ij} = \epsilon_m\left[E_iE_j - \frac{1}{2}E_kE_k\delta_{ij}\right],$$

where ϵ_m is the permittivity of the concerned medium.

Further we have uniform normal electric field far from the surface, which gives

$$\Phi_z \to E_0 \quad \text{as} \quad z \to \infty, \tag{12}$$

where E_0 is the basic uniform normal applied electric field and at the free surface $(z = h(x,t))$ as another boundary condition.

The above equations are quite general regarding various coefficients (k_T, κ, μ, σ, etc.). It is well known that temperature variation in the fluid can cause dramatic changes in the above coefficients, but approximations can be made depending on the type of the problem being examined. In the foregoing analysis, we have assumed the variation of surface tension as

$$\sigma(T) = \sigma_0 - \gamma(T - T_g), \tag{13}$$

where σ_0 is the surface tension at T_g, the temperature in the gas phase, which is taken as the reference temperature and $\gamma = -\partial\sigma/\partial T|_{T=T_g}$ is a positive constant for most common fluids. The assumption of linear variation of surface tension with temperature is very much compatible with the experimental data. Apart from water [8,10] there are many liquids [11] which follow the linear variation of surface tension with the temperature scales. For example, the molten tin (Sn) in the range of 520–1670 K and molten zirconium (Zr) in the range 2000–2250 K follow the law

$$\sigma(T) = 561.6 - 0.103(T - 505)\ \text{m Nm}^{-1} \quad \text{and}$$
$$\sigma(T) = 1543 - 0.66(T - 2128)\ \text{m Nm}^{-1},$$

respectively. It is to be noted here that both the liquids are highly conducting.

To express the governing equations and boundary conditions in non-dimensional form, we shall assume two length scales l_0 and h_0 as the characteristic measure for the length in longitudinal and transverse direction, respectively; l_0 may be assumed as one wave length and h_0 is the mean depth of the film, which gives $l_0 \gg h_0$. Further to measure the transverse length in the vacuum, which extend to infinity, h_0 is not a proper scale, so we shall consider l_0 as the measure of the transverse length in the vacuum. The Nusselt velocity $u_0 = gh_0^2\sin\theta/3\nu$ will be assumed as the characteristic velocity along the longitudinal direction. We define the dimensionless quantities as

$$\begin{aligned} &x = l_0x^*, \quad h = h_0h^*, \quad z = h_0z^* \ (\text{in liquid}),\\ &z = (l_0/h_0)\zeta \ (\text{in vacuum}), \quad t = (l_0/u_0)t^*, \quad u = u_0u^*,\\ &v = (h_0/l_0)u_0v^*, \quad p = \rho u_0^2p^*, \quad T = T_g + T^*(T_H - T_C),\\ &\Phi = E_0h_0\Phi^*. \end{aligned} \tag{14}$$