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Three dimensional mixed convection in plane symmetric-sudden expansion: Symmetric flow regime

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ABSTRACT

Three-dimensional simulations of laminar buoyancy assisting mixed convection in a vertical duct with a plane symmetric sudden expansion are presented to illustrate the effects of the buoyancy assisting force and the duct's aspect ratio on the flow and heat transfer. This geometry and flow conditions appear in many engineering applications, but 3-D heat transfer results have not appeared in the literature. This study focuses on the regime where the flow and thermal fields are symmetric in this geometry. The buoyancy force is varied by changing the heat flux on the stepped walls that are downstream from the sudden expansion, and the duct's aspect ratio is varied by changing the width of the duct while keeping the expansion ratio constant. Results are presented for duct's aspect ratio of 4, 8, 12, 16, and ∞ (2-D flow), and for wall heat fluxes between 5–35 W/m^2. The Reynolds number and the range of wall heat flux are selected to insure that the flow remains laminar and symmetric in this geometry and reverse flow does not develop at the exit section of the duct. Results for the velocity, temperature, and the Nusselt number distributions are presented, and the effects of the buoyancy force and the duct's aspect ratio on these results are discussed.

1. Introduction

The separation of flow and its subsequent reattachment due to sudden changes in geometry is a common phenomenon that occurs in many engineering applications such as in electronic cooling equipment, cooling of turbine blades, combustion chambers, and many other heat exchanging devices. Studies for isothermal laminar flow [1–6] in plane symmetric sudden expansion have shown that the flow is steady and symmetric for Reynolds number lower than a critical value, and asymmetric and steady for Reynolds number higher than the critical value. These studies also show that the critical Reynolds number increases with decreasing the aspect ratio and also with decreasing the expansion ratio of the duct. Tsui and Shu [7] and Alimi et al. [8] reported results for 2-D laminar mixed convection for the asymmetric flow regime. Thiruvengadam et al. [9] simulated the bifurcated 3-D forced convection, and in [10] they reported the effect of the buoyancy force and duct's aspect ratio on that asymmetric flow regime. The introduction of low levels of heating at the stepped walls (buoyancy assisting) caused the level of asymmetry in the flow to decrease for a fixed Reynolds number. The asymmetry ceases to exist in the flow and thermal fields, and a symmetric mixed convection flow regime develops at and beyond a critical level of buoyancy assisting force. The critical value of the buoyancy assisting force (or the wall heat flux) increases as the duct's aspect ratio increases thus reaching its maximum value for a duct with infinite aspect ratio (i.e. 2-D flow) for a fixed Reynolds number. For example, for the geometry and flow conditions that are considered in this study, i.e. duct's expansion ratio of two and Reynolds number of 800, symmetric buoyancy assisting mixed convection flow regime develops for wall heat flux larger than 6.21 W/m^2 for any of the aspect ratios. To the authors knowledge the effects of buoyancy force and duct's aspect ratio on the 3-D laminar symmetric mixed convection flow regime that develops in this geometry has not appeared in the literature and that motivated the present study.

2. Problem and solution procedure

This work, which focuses on the symmetric flow regime, is an extension of the work by Thiruvengadam et al. [9,10] where the asymmetric flow regime was examined. A schematic of the vertical plane symmetric sudden expansion geometry and the computational domain that is used in this simulation is presented in Fig. 1. The upstream duct height (h) is 0.02 m and the downstream duct height (H) is 0.04 m, respectively, resulting in expansion ratio (ER = H/h) of 2. The step height (S) is maintained at 0.01 m and the duct's width (W) is varied to examine the effect of upstream aspect ratio (AR = W/h) on the results. The origin of the coordinate system is located at the bottom corner of the sudden expansion as shown

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Nomenclature

AR	Upstream aspect ratio = W/h
ER	Expansion ratio = H/h
H	Duct height downstream from the sudden expansion
h	Duct height upstream from the sudden expansion
k	Thermal conductivity
L	Half width of the duct = $W/2$
Nu	Local Nusselt number = $q_w S/k(T_w - T_0)$
$Nu_{step,avg}$	Average Nusselt number on the stepped wall = $q_w S/k(T_{step,avg} - T_b)$
q_w	Wall heat flux = $-k\partial T/\partial n$ at the wall
Re	Reynolds number = $2\rho u_0 h/\mu$
S	Step height
T	Temperature
T_b	Bulk fluid temperature = $\frac{\int_0^W \int_0^H u(x,y,z)T(x,y,z)\mathrm{d}y\mathrm{d}z}{\int_0^W \int_0^H u(x,y,z)\mathrm{d}y\mathrm{d}z}$
T_w	Local wall Temperature
T_0	Inlet fluid temperature
$T_{step,avg}$	Average stepped wall temperature = $\frac{1}{L}\int_0^L T_{w,step}\mathrm{d}z$
$T_{side,avg}$	Average side wall temperature = $\frac{1}{H}\int_0^H T_{w,\,side}\mathrm{d}y$
u	Velocity component in the x-coordinate direction
u_0	Average inlet velocity
v	Velocity component in the y-coordinate direction
W	Width of the duct
w	Velocity component in the z-coordinate direction
x	Streamwise coordinate
x_u	Locations where the streamwise velocity gradient is zero ($\partial u/\partial y_{\text{at wall}} = 0$)
y	Transverse coordinate
z	Spanwise coordinate

Greek symbols

β	Volumetric coefficient of thermal expansion
μ	Dynamic viscosity
ρ	Density

in Fig. 1. The directions of the streamwise (x), transverse (y), and spanwise (z) coordinates are shown in that figure. The length of the computational domain is 1.0 m downstream and 0.02 m upstream of the sudden expansion, respectively, i.e. $-2 \leqslant x/S \leqslant 100$. This choice is made to insure that the flow at the inlet section of the duct ($x/S = -2$) is not affected by the sudden expansion in geometry, and reverse flow does not develop at the exit section of the duct (i.e. starved flow condition at the higher wall heat flux) and can be treated as fully developed. The governing equations for steady laminar, incompressible, three-dimensional, buoyancy assisting mixed convection with constant properties are formulated for the continuity, momentum, and energy conservation. Thermal buoyancy effects are modeled using the Boussinesq approximation. Details of the governing equations can be found in Li and Armaly [11] who studied laminar mixed convection

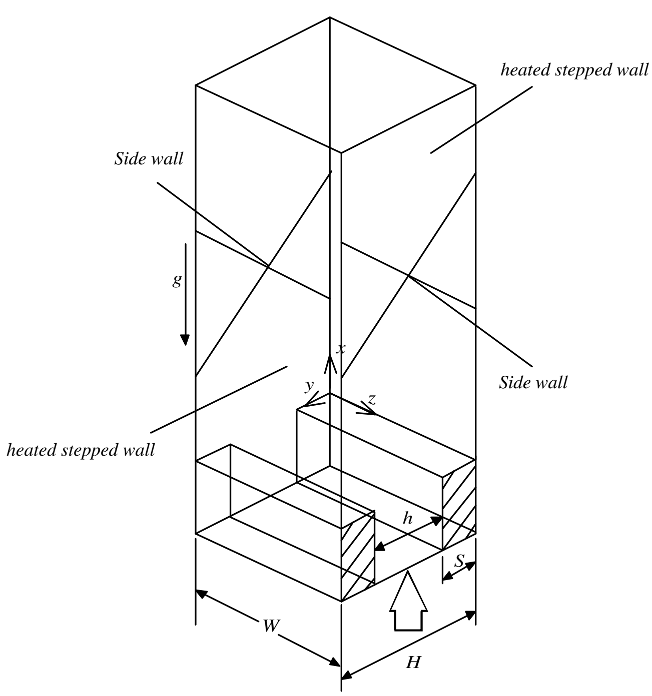

Fig. 1. Schematic of the computational domain.

effects in 3-D backward facing step and these equations will not be re-stated here due to space limitations. The full elliptic 3-D coupled governing equations are solved numerically using finite volume method to simulate the flow and the temperature fields in this computational domain.

The physical properties are treated as constants and evaluated for air at the inlet temperature of $T_0 = 20\,^{\circ}\mathrm{C}$ (i.e. density (ρ) is 1.205 kg/m^3, specific heat (C_p) is 1005 J/kg K, dynamic viscosity (μ) is 1.81×10^{-5} kg/m s, thermal conductivity (k) is 0.0259 W/m K, and volumetric coefficient of thermal expansion (β) is 0.00341 1/K). Inlet flow (at $x/S = -2$, $1 \leqslant y/S \leqslant 3$, for all z) is considered to be isothermal ($T_0 = 20\,^{\circ}\mathrm{C}$), steady, and fully developed. The distribution for the isothermal fully developed streamwise velocity component (u) in rectangular duct that was used in this study is given by Shah and Bhatti [12] and is not repeated here due to space limitations. The other velocity components (transverse (v) and spanwise (w)) are set to be equal to zero at that inlet section. The no-slip boundary condition (zero velocities) is applied to all of the wall surfaces. Uniform and constant wall heat flux (q_w) is specified at the two stepped walls (at $y/S = 0$ and 4, $0 \leqslant x/S \leqslant 100$, for all z), while other walls (side walls) are treated as adiabatic surfaces. The magnitude of that wall heat flux is varied between 5–35 W/m^2 while keeping the Reynolds number ($Re = 2\rho u_0 h/\mu$, where u_0 is the average inlet velocity) fixed at 800 in order to investigate the effects of the buoyancy assisting force on the flow and heat transfer. Similarly, the magnitude of the wall heat flux is fixed at 15 W/m^2, and the Reynolds number is fixed at 800 while the aspect ratio is varied in order to investigate its effect on the flow and heat transfer. Due to the symmetry of the flow and thermal fields in the spanwise direction for the stated conditions in this geometry, the width of the computational domain is chosen as half of the actual width of the duct, $L = W/2$, and symmetry boundary conditions are applied at the center width of the duct, i.e. at $z = L$, $w = 0$, and the gradient of all the other quantities with respect to z are set equal to zero. Fully developed flow and thermal conditions are imposed at the exit section (at $x/S = 100$, for all y and z) of the calculation domain.

Numerical solution of the governing equations and boundary conditions are performed by utilizing the commercial computational fluid dynamics (CFD) code FLUENT 6.2. The mesh is generated using FLUENT's preprocessor GAMBIT. Hexahedron volume elements are used in the simulation. The residual sum for each of the conserved variables is computed and stored at the end of each iteration, thus recording the convergence history. The convergence