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Laminar natural convection in a square cavity: Low Prandtl numbers and large density differences

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ABSTRACT

Steady natural convection at low Prandtl numbers caused by large density differences in a square cavity heated through the side walls is investigated numerically and theoretically. An appropriate dimensionless parameter characterizing the density differences of the working fluid is identified by the Gay-Lussac number. The Boussinesq assumption is achieved when the Gay-Lussac number tends to zero. The Nusselt number is derived for the ranges in Rayleigh number $10 \leqslant Ra \leqslant 10^8$, in Prandtl number $0.0071 \leqslant Pr \leqslant 7.1$ and in Gay-Lussac number $0 \leqslant Ga < 2$. The effects of the Rayleigh, Prandtl and Gay-Lussac numbers on the Nusselt number are discussed on physical grounds by means of a scale analysis. Finally, based on physical arguments, a heat transfer correlation is proposed, valid for all Prandtl and Gay-Lussac number ranges addressed.

1. Introduction

Natural convection in cavities has been intensively studied in the literature due to its relevance to many fields of science and technology such as geophysics, nuclear reactor systems, energy storage and foundry processes.

There are numerous studies in the literature regarding natural convection in cavities, a considerable amount of which is reviewed by Ostrach [1]. In particular, rectangular and square cavities are the most frequently studied due to their many thermo-fluid features, such as recirculation and stagnation regions, boundary layers, jet deflection, and thermal entrainment.

In the case of natural convection in cavities due only to temperature differences, in absence of both heat generation and concentration gradients, three main configurations have been considered in the literature:

(1) natural convection in horizontal layers heated through the top and bottom walls;
(2) sideways heating of an initially stratified fluid layer;
(3) natural convection in enclosures heated through the side walls.

The square cavity has been regarded in the literature as the most suitable case for the validation of numerical codes for thermal analysis and for physical understanding of natural convection in enclosures.

De Vahl Davis [2] provided a well known set of benchmark solutions for steady natural convection of air in a horizontally heated square cavity for Rayleigh numbers up to 10^6. Le Quéré [3] extended the analysis up to $Ra = 10^8$.

The account of possible interactions between the fluid in an enclosure and its surroundings can also be of practical interest. The influence of participating walls has been analysed for instance by Costa [4], while the effect of solids located at the corners of the cavity has been investigated by Costa et al. [5]. However, in most of the papers [1] conductive walls are not included.

The working fluids analysed in the literature have been mainly air and water. In addition, due to the interest in foundry processes, crystal-growing and nuclear reactor systems, liquid metals have also been studied.

Braunsfurth et al. [6] presented numerical and experimental temperature profiles corresponding to laminar natural convection of liquid gallium in a rectangular cavity heated through the side walls. For the same problem, a simplified model was proposed by Graebel [7]: the heat transfer results have been derived analytically for the Prandtl number range from about 0.05 up to infinity. Lage and Bejan [8] studied laminar natural convection in a square enclosure heated through the side walls for $0.01 \leqslant Pr \leqslant 10$ and $10^2 \leqslant Ra \leqslant 10^{11}$ and addressed the influence of the Prandtl number on the heat transfer. A similar problem has been analysed for $0.011 \leqslant Pr \leqslant 0.054$ by Saravanan and Kandaswamy [9]: they observed a significant effect of a variable thermal conductivity on the heat transfer through the cavity. For liquid gallium, significant differences were also evident in a comparison between 2-D and 3-D numerical predictions carried out by Derebail and Koster [10].

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Nomenclature

c_p	specific heat at constant pressure
F'_B	buoyancy forces per unit depth
F'_I	inertial forces per unit depth
F'_v	viscous forces per unit depth
g	acceleration of gravity
Ga	Gay-Lussac number, $Ga = \beta_o \Theta$
h'	specific enthalpy
L	width of the cavity
$\mathbf{n}$	outward unit normal vector
Nu	average Nusselt number, Eq. (24)
Nu_y	local Nusselt number, Eq. (23)
$Nu_{y,max}$	maximum value of local Nusselt number
$Nu_{y,min}$	minimum value of local Nusselt number
p'	pressure
p	dimensionless pressure, $p = (p' + \rho_o g y')/P$
P	reference pressure, $P = \rho_o U^2$
Pr	Prandtl number, $Pr = \upsilon_o/\alpha_o$
Q'	heat flux per unit depth through an isothermal side of the cavity
q''	specific heat flux per unit depth
Ra	Rayleigh number, $Ra = g\beta_o \Theta L^3/(\alpha_o \upsilon_o)$
T'	temperature
T'_o	reference temperature, $T'_o = (T'_h + T'_c)/2$
T	dimensionless temperature, $T = (T'_o - T')/\Theta$
u', v'	Cartesian velocity components
u, v	dimensionless Cartesian velocity components, $u = u'/U$ and $v = v'/U$
U	reference velocity, $U = (g\beta_o L\Theta)^{1/2}$
V	characteristic vertical velocity on the thermal layer
x', y'	Cartesian co-ordinates
x, y	Cartesian dimensionless co-ordinates, x'/L and y'/L

Greek symbols

α	thermal diffusivity
β	volumetric coefficient of thermal expansion
δ_T	thermal layer thickness
λ	thermal conductivity
μ	dynamic viscosity
υ	kinematic viscosity
Θ	characteristic temperature difference, $\Theta = T'_h - T'_c$
ρ	density
Π_N	dimensionless parameter, Eq. (42)

Subscripts

c	cold wall
h	hot wall
o	at the reference temperature T'_o
r	at the reference temperature T'_r
w	wall
adv	advection
cond	conduction

In connection with advanced engineering applications in nuclear reactor systems, recent studies on natural convection in enclosures at low Prandtl numbers address the effects of magneto-hydrodynamic interactions and volumetric heating. Some representative studies can be found in [11–15].

The whole quoted papers [1–15] deal with the Boussinesq approximation. However, in many practical applications, the natural convection is driven by high temperature differences and, for these cases, the Boussinesq approach may be too restrictive because of the strong variations in the thermophysical properties of the working fluid. This is the case for instance of foundry processes. For these applications the study of natural convection in enclosures due to large temperature differences could provide important considerations in the design, both for more efficient operations and for higher quality manufactures.

In the literature, only a few studies have been undertaken to examine the influence of the variability of thermophysical properties on laminar natural convection in cavities.

In the case of heating through the side walls, variable physical properties have been considered by Zhong et al. [16]. They observed significant variable properties effects on the heat transfer and also suggested a limit of validity of the Boussinesq approximation and a heat transfer correlation.

Becker and Braack [17] provided numerical predictions for the case of laminar natural convection of a weakly compressible ideal gas (air) in a square cavity heated through the side walls. The same problem, but for a fully compressible ideal gas, has been analysed by Vierendeels et al. [18] and by Darabandi and Hosseinizadeh [19]. A very good agreement can be observed among the predictions in [17–19]. For the same problem, Paillere et al. [20] found a very good agreement between the results obtained by the weakly compressible and the fully compressible model. In these papers [17–20] the Mach number is always very low; for example, the highest local Mach number predicted in [20] was about of 10^{-4}.

It may be concluded from reviewing the literature that there is a lack of information on the influence of low and very low Prandtl numbers and of large density differences due to large temperature differences on heat transfer due to natural convection.

The present paper deals with these latter aspects. The analysis has been carried out for the case of laminar flow in a square cavity heated through the side walls.

The dimensionless parameter characterizing the density differences of the working fluid has been identified as the Gay-Lussac number. Its influence on the Nusselt number has been derived over its entire physical domain, where the limiting cases lead to the Boussinesq assumption and to extreme density variation, respectively.

Also, the influence of the Prandtl number on Nusselt number has been examined for $0.0071 \leqslant Pr \leqslant 7.1$. The Rayleigh number studied here ranges from 10 up to 10^8.

The governing equations were treated by the finite volume CFD commercial package ANSYS CFX-10.0 [21].

The numerical procedure has been validated by comparison against the results provided by de Vahl Davis [2], Le Quéré [3], Lage and Bejan [8] and Wan et al. [22].

The Nusselt number has been derived for the above ranges of *Ra*, *Pr* and *Ga* numbers.

Finally, a correlation relating the laminar Nusselt number to the above ranges of parameters has been proposed. Its development is based on physical arguments based on the conservation of momentum and thermal energy.

2. Mathematical formulation

The system to be considered (Fig. 1) is a two-dimensional square cavity of width L, where the two vertical walls are kept at different temperature, T'_h and T'_c. Zero heat flow is assumed at the top and bottom walls. The walls are rigid and impermeable, and no-slip boundary conditions are imposed at the boundaries.