Available online at www.sciencedirect.com
ScienceDirect
International Journal of Heat and Mass Transfer 51 (2008) 3103–3112
International Journal of HEAT and MASS TRANSFER
www.elsevier.com/locate/ijhmt

Meshless method based on the local weak-forms for steady-state heat conduction problems

Wu Xue-Hong, Tao Wen-Quan *

State Key Laboratory of Multiphase Flow in Power Engineering, School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi 710049, PR China

Received 23 March 2007; received in revised form 22 August 2007
Available online 23 October 2007

Abstract

In this article, the meshless local Petrov–Galerkin (MLPG) method is applied to compute two steady-state heat conduction problems of irregular complex domain in 2D space. The essential boundary condition is enforced by the transformation method, and the MLS method is used for interpolation schemes. A numerical example that has analytical solution shows the present method can obtain desired accuracy and efficiency. Two cases in engineering with irregular boundary are computed to validate the approach by comparing the present method with the finite volume method (FVM) solutions obtained from a commercial CFD package FLUENT 6.3. The results show that the present method is in good agreement with FVM. It is expected that MLPG method (which is a truly meshless) is very promising in solving engineering heat conduction problems within irregular domains.

Keywords: Meshless method; Local Petrov–Galerkin method; MLS; Heat conduction

1. Introduction

The finite volume method (FVM) and finite element method (FEM) have been widely applied to solve the practical engineering problems. It is well-known that these methods depend strongly on the mesh properties. However, to compute problems with irregular complex geometries by using these methods, mesh generation is a far more time-consuming and expensive task than solution of the partial differential equations (PDEs), particularly in 3D cases. Owing to the difficulty of FVM and FEM in the mesh generation, a new numerical method, meshless method (also called meshfree method), has been developed fast in the recent years. In the following a brief review is presented. The smoothed particle hydrodynamics method that was initially used for modelling astrophysical phenomena is now widely used in such complicated phenomena as explosion and underwater shock problems [1]. The earlier research works on SPH may be found in Lucy [2] and Monaghan [3]. Diffuse approximation method (DAM) [4] is closely related to the moving least-squares method, which has been used in the framework of a Galerkin formulation to develop the diffuse element method (DEM) [5]. The element free Galerkin (EFG) method [6] is based on the DEM and widely used in many mechanics problems. The reproducing kernel particle method (RKPM) [7] is to improve the SPH approximation to satisfy consistency requirements using a corrections function, and it has been used in nonlinear and large deformation problems of solid mechanics. It should be noted that most of these methods are not really meshless method, since they need to use a background mesh for the numerical integration. The finite point method (FPM) [8] and MLPG method [9,10] are both truly meshless methods. The FPM uses a non-element interpolation scheme-weighted least square and has no integration required. But this method is based on the point collocation, and the solution results are very sensitive to the selection of the collocation points. The MLPG is based

* Corresponding author. Tel.: +86 29 82669106; fax: +86 29 82669738.
E-mail address: wqtao@mail.xjtu.edu.cn (W.-Q. Tao).

0017-9310/$ - see front matter © 2007 Published by Elsevier Ltd.
doi:10.1016/j.ijheatmasstransfer.2007.08.021

Nomenclature

$a(x)$, $\mathbf{a}(\mathbf{x})$	coefficient of basis function and its vector form
h	heat transfer coefficient
n	unit normal vector outward to the boundary
$p(x)$, $\mathbf{p}(\mathbf{x})$	basis function and its vector form
q	heat flux on the boundary
r	size of support for the weight functions
T	temperature
$T^h(x_I)$	trial functions
$\widehat{T}^I$	fictitious nodal values
$w_I(x)$	weight function
x, y	spacial coordinates

Greek symbols

λ	thermal conductivity
$\phi(x)$, $\mathbf{\Phi}(\mathbf{x})$	shape function and its vector form
$\dot{\Phi}$	heat source
Γ_1, Γ_2, Γ_2	boundaries
Ω	problem domain

Subscripts and superscripts

a	analytical
h	approximate
I, J, K	node indices
m	number of terms
M	total number of nodes
N	number of nodes
num	numerical

on the weak form computed over a local sub-domain and easy to deal with different boundary value problems. Detailed introductions of the MLPG method can be found in Atluri and Shen [11].

A number of meshless methods have been developed by different authors to solve heat transfer and fluid flow problems [12–15]. The focus of the present study is concentrated on the heat conduction problems. In the following a brief review related to the present study is presented. Cleary and Monaghan [16] and Chen et al. [17] employed smoothed particle hydrodynamics method to solve unsteady-state heat conduction problem. Singh and his colleagues used EFG method to solve a number of heat conduction problems, including the nonlinear heat conduction [18], 2D fins [19], 3D steady-state [20] and transient [21] heat conduction problems and composite heat transfer problems [22],and their investigated results show that the EFG results are more accurate than the FEM results [18]. Liu et al. [23] used meshless weighted least-squares (MWLS) method to solve steady- and unsteady-state heat conduction problems. Tan et al. [24] applied least-squares collocation meshless method to solve coupled radiative and heat conduction problems. Sadat et al. [25] used DAM to solve a two-dimensional heterogeneous heat conduction problem. Qian et al. [26] applied MLPG method to compute three-dimensional transient heat conduction problem. Sladek et al. [27,28] applied MLPG method to solve the heat conduction problem in an anisotropic medium.

From above brief review on meshless method application in solving heat conduction problems, we can see that previous researchers have focused mainly on using EFG, SPH and MLPG method. However, the EFG method needs a background mesh for the integrals in the weak form, hence it is not really meshless method; the SPH and DAM and MLWS method are built on the collocation point schemes, for which the selection of the collocation point are important, and the numerical accuracy goes down near the boundary. MLPG method is a truly meshless method; it offers a lot of flexibility to deal with problems of different boundary conditions. A wide range of problems have been investigated by Atluri and his coauthors using MLPG method. Almost all of the previous works limited to heat conduction problems of regular domain. However, many problems in engineering are in irregular domain, and FVM and FEM are difficult to describe accurately boundaries of the irregular domain unless the mesh is very fine, or special grid generation method is adopted which is usually time-consuming. Meshless methods can overcome this difficulty because they do not need mesh. Meshless methods distribute arbitrarily scattering points in the problem domain, so they will have more advantages in solving problems with irregular domain than FVM and FEM. So in the present paper, we apply MLPG method to compute two steady-state heat conduction problems of irregular domain encountered in engineering.

The following discussion begins with implementation of local Petrov–Galerkin method for heat conduction problem in Section 2. The results of numerical examples and discussion are presented in Section 3. The paper ends with conclusions in Section 4.

2. Implementation of local Petrov–Galerkin method for heat conduction problem

As other numerical simulation methods such as FVM, the MLPG method needs some kind of interpolation schemes and discretization methods to generate the algebraic equations, which can be solved numerically. There are a number of local interpolation schemes, such as moving least-square (MLS) approximation, partition of unity