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A general bioheat transfer model based on the theory of porous media

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Abstract

A volume averaging theory (VAT) established in the field of fluid-saturated porous media has been successfully exploited to derive a general set of bioheat transfer equations for blood flows and its surrounding biological tissue. A closed set of macroscopic governing equations for both velocity and temperature fields in intra- and extravascular phases has been established, for the first time, using the theory of anisotropic porous media. Firstly, two individual macroscopic energy equations are derived for the blood flow and its surrounding tissue under the thermal non-equilibrium condition. The blood perfusion term is identified and modeled in consideration of the transvascular flow in the extravascular region, while the dispersion and interfacial heat transfer terms are modeled according to conventional porous media treatments. It is shown that the resulting two-energy equation model reduces to Pennes model, Wulff model and their modifications, under appropriate conditions. Subsequently, the two-energy equation model has been extended to the three-energy equation version, in order to account for the countercurrent heat transfer between closely spaced arteries and veins in the circulatory system and its effect on the peripheral heat transfer. This general form of three-energy equation model naturally reduces to the energy equations for the tissue, proposed by Chato, Keller and Seiler. Controversial issues on blood perfusion, dispersion and interfacial heat transfer coefficient are discussed in a rigorous mathematical manner.

Keywords: Bioheat transfer; Dispersion; Porous media; Volume averaging

1. Introduction

A number of bioheat transfer equations for living tissue have been proposed since the landmark paper by Pennes [1] appeared in 1948, in which the perfusion heat source was introduced. Although Pennes model is often adequate for roughly describing the effect of blood flow on the tissue temperature, some serious shortcomings exist in his model due to its inherent simplicity, as pointed out by Wulff [2], namely, assuming uniform perfusion rate without accounting for blood flow direction, neglecting the important anatomical features of the circulatory network system such as countercurrent arrangement of the system, and choosing only the venous blood stream as the fluid stream equilibrated with the tissue.

In order to overcome these shortcomings, a considerable number of modifications have been proposed by various researchers. Wulff [2] and Klinger [3] considered the local blood mass flux to account the blood flow direction, while Chen and Holmes [4] examined the effect of thermal equilibration length on the blood temperature and added the dispersion and microcirculatory perfusion terms to the Klinger equation.

All foregoing papers concerned mainly with the cases of isolated vessels and the surrounding tissue. The effect of countercurrent heat transfer between closely spaced arteries and veins in the tissue must be taken into full consideration when the anatomical configuration of the main supply artery and vein in the limbs is treated. Following the experimental study conducted by Bazett and his colleagues [5,6], Scholander and Krog [7] and Mitchell and

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Nomenclature

A	surface area
A_{int}	interface between the fluid and solid
a_{f}	specific surface area
b_{ij}	Forchheimer tensor
c_p	specific heat at constant pressure
h_{f}	interfacial heat transfer coefficient
k	thermal conductivity
K_{ij}	permeability tensor
n_j	unit vector pointing outward from the fluid side to solid side
p	pressure
S_{m}	metabolic reaction rate
T	temperature
u_i	velocity vector
V	representative elementary volume
x, y	Cartesian coordinates
α	thermal diffusivity
ε	porosity
ν	kinematic viscosity
ρ	density
ω	perfusion rate
ω'	net filtration rate

Special symbols

$\tilde{\phi}$	deviation from intrinsic average
$\langle\phi\rangle$	volume average
$\langle\phi\rangle^{\text{f,s,a,v}}$	intrinsic average

Subscripts and superscripts

a	artery
dis	dispersion
f	fluid
s	solid
v	vein

Myers [8] investigated such an effect and successfully demonstrated that the countercurrent heat exchange reduces heat loss from the extremity to the surroundings, which could be quite significant due to a large surface to volume ratio. Keller and Seiler [9] established a bioheat transfer model equation to include the countercurrent heat transfer, using a one-dimensional configuration for the subcutaneous tissue region with arteries, veins and capillaries. Weinbaum and Jiji [10] proposed a new model, which is based on some anatomical understanding, considering the countercurrent arterio–venous vessels. As pointed out by Roetzel and Xuan [11], the model may be useful in describing a temperature field in a single organ, but would not be convenient to apply to the whole thermoregulation system. Excellent reviews on these bioheat transfer equations may be found in Chato [12] and Charny [13].

Khaled and Vafai [14] and Khanafer and Vafai [15] stress that the theory of porous media is most appropriate for treating heat transfer in biological tissues since it contains fewer assumptions as compared to different bioheat transfer equations. Roetzel and Xuan [11] and Xuan and Roetzel [16] exploited the volume averaging theory (VAT) previously established for the study of porous media (e.g. Cheng [17], Nakayama [18]), to formulate a two-energy equation model accounting for the thermal non-equilibrium between the blood and peripheral tissue. In their model, the perfusion term is replaced by the interfacial convective heat transfer term. This point should be examined since the interfacial convective heat transfer is different from perfusion heat transfer. Naturally, the former takes place even in the absence of the latter.

In this study, we present a rigorous mathematical development based on the volume averaging theory, so as to achieve a complete set of the volume averaged governing equations for bioheat transfer and blood flow. Most shortcomings in existing models will be overcome. We start with the case of isolated blood vessels and the surrounding tissue, to establish a two-energy equation model for the blood and tissue temperatures. We shall identify the terms describing the blood perfusion and dispersion in the resulting equation and revisit Pennes model, Wulff model and their modifications.

Subsequently, the two-energy equation model is extended to the three-energy equation model, so as to account for the effect of countercurrent heat transfer between closely spaced arteries and veins in the blood circulatory system. In this model, three individual temperatures are assigned for the arteries, veins and tissue. We shall examine the Keller and Seiler model [9] and Chato model [12] for the microcirculation as well as the model proposed by Xuan and Roetzel [16] for simulation of transient response of the human limb to an external stimulus. Controversial issues on blood perfusion, dispersion and heat transfer coefficient will be discussed in a rigorous mathematical manner.

2. Volume averaging procedure

In an anatomical view, three compartments are identified in the biological tissues, namely, blood vessels, cells and interstitium, as illustrated in Fig. 1. The interstitial space can be further divided into the extracellular matrix and the interstitial fluid. However, for sake of simplicity, we divide the biological tissue into two distinctive regions, namely, the vascular region and the extravascular region (i.e. cells and the interstitium) and treat the whole anatomical structure as a fluid-saturated porous medium, through which the blood infiltrates. The extravascular region is regarded as a solid matrix (although the extravascular fluid