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The heat/mass transfer analogy for a simulated turbine endwall

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Abstract

Heat transfer measurements in gas turbine cascades are often difficult because of thin boundary layers, complex secondary flows, and large variation in local heat transfer rates. Thus mass transfer techniques have often been used as an alternative method, the heat transfer coefficients being then calculated from the heat/mass transfer analogy.

To ensure confidence in the quantitative conversion to the heat transfer coefficients from the mass transfer results, evaluation of the analogy factors is crucial. The present paper examines the validity of the heat/mass transfer analogy, evaluating the analogy factors on a simulated turbine endwall, with separate heat and mass transfer experiments with equivalent flow and geometric conditions. The Nusselt numbers, determined from the heat transfer experiments with a constant wall temperature boundary condition are compared to Sherwood numbers from the mass transfer experiments employing a constant wall concentration boundary condition to evaluate the heat/mass transfer analogy.

Keywords: Heat/mass transfer analogy; Naphthalene sublimation; Thermal boundary layer; Constant temperature; Constant concentration; Turbine endwall

1. Introduction

Mass transfer experiments have been extensively used to determine heat transfer coefficients using the heat/mass transfer analogy. Local mass transfer results can be obtained under laboratory conditions with a high resolution and precision in a short time period. Mass transfer experiments are free from conduction and radiation errors which are inherent in heat transfer studies. However, when precise heat transfer results are required, detailed local heat/mass transfer analogy factors may be required to convert the mass transfer data into heat transfer results.

The heat/mass transfer analogy is derived from the conservation equations of momentum, energy and concentration of a constant property fluid by Nusselt [1] and Schmidt [2]. A heat/mass transfer analogy implies that heat transfer results can be converted from the mass transfer results under equivalent experimental conditions and vice-versa. This follows from the similarity of the equations governing heat and mass transfer.

The non-dimensional heat transfer equation

$$\frac{\mathrm{D}\theta}{\mathrm{D}\tau} = \frac{1}{RePr}\frac{\partial}{\partial \widehat{x}_i}\left(\left(1+\frac{\epsilon}{\nu}\frac{Pr}{Pr_\mathrm{t}}\right)\frac{\partial \theta}{\partial \widehat{x}_i}\right) \tag{1}$$

and the non-dimensional mass transfer equation

$$\frac{\mathrm{D}m}{\mathrm{D}\tau} = \frac{1}{ReSc}\frac{\partial}{\partial \widehat{x}_i}\left(\left(1+\frac{\epsilon}{\nu}\frac{Sc}{Sc_\mathrm{t}}\right)\frac{\partial m}{\partial \widehat{x}_i}\right) \tag{2}$$

are very similar. Hence, if the boundary conditions are equivalent for a given geometry, if Pr is equal to Sc, and Pr_t is equal to Sc_t, then 'θ' in Eq. (1) and 'm' in Eq. (2) have the same variations. This essentially describes the heat/mass transfer analogy. It should, however, be noted that the Prandtl number in a heat transfer experiment is typically different from the Schmidt number in a mass transfer experiment. Therefore, the usefulness of the heat/mass

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Nomenclature

AR	inlet/exit area ratio of the cascade = 2.72
$C_l, (C_L)$	characteristic length, $C_l = 184$ mm in present study
C_x	axial chord length of blade, =130 mm in present study
d	distance from a blade leading edge
D_{naph}	mass diffusion coefficient
F	local analogy factor, $F = Nu/Sh$
H	height of blade = 457 mm
h	heat transfer coefficient
h_{m}	mass transfer coefficient
m	dimensionless mass fraction
M_{naph}	molecular mass for naphthalene, $M_{\text{naph}} = 129.17$ kg/kmol
n	power index used in heat/mass transfer analogy
n	the distance in the direction normal to the wall
Nu	Nusselt number, $Nu = h \cdot C_l/k$
P	pitch of blade, =138 mm
$p_{\text{v,w}}$	naphthalene vapor pressure at the wall
Pr	Prandtl number, $Pr = \nu/\alpha$
Pr_{t}	turbulent Prandtl number, $Pr_{\text{t}} = \epsilon/\epsilon_H$
q_{w}	heat flux from the wall
R	gas constant for naphthalene$(C_{10}H_8), R = 0.06487$ J/g K
$\overline{R}$	universal gas constant
Re_{ex}	exit Reynolds number, $= \rho U_{\text{ex}} C_l/\mu$
S	curvilinear coordinate on the blade
S_{p}	curvilinear coordinate on the pressure surface, cf. Fig. 8
S_{s}	curvilinear coordinate on the suction surface, cf. Fig. 8
Sc	Schmidt number, $Sc = \nu/D_{\text{naph}}$
Sc_{t}	turbulent Schmidt number, $Sc_{\text{t}} = \epsilon/\epsilon_M$
Sh	Sherwood number, $Sh = h_{\text{m}} \cdot C_l/D_{\text{naph}}$
St	Stanton number, $St = \frac{Nu}{RePr} = h/\rho c_p U_\infty$
St_{m}	mass transfer Stanton number, $St_{\text{m}} = \frac{Sh}{ReSc} = h_{\text{m}}/U_\infty$
T_{w}	wall temperature
T_∞	free stream temperature
T_{aw}	adiabatic wall temperature
$T_{\text{n,w}}$	surface temperature of naphthalene
Tu	turbulence intensity, $Tu = \frac{\sqrt{\overline{u'^2}}}{U_\infty}$
U_∞	mainstream inflow velocity in wind tunnel
x	coordinate in blade chord direction
x_i	coordinate along the inflow direction, cf. Fig. 8
y	coordinate traverse to the blade chord direction
Z	coordinate in spanwise direction of cascade, $Z = 0$ at the top

Greek symbols

$\delta\tau$	time duration of data run
δt	sublimation depth of naphthalene
ϵ	turbulent momentum diffusivity
ϵ_H	turbulent thermal diffusivity
ϵ_M	turbulent mass diffusivity
ρ_{s}	density of solid naphthalene
$\rho_{\text{v},\infty}$	naphthalene vapor density in free stream
$\rho_{\text{v,w}}$	naphthalene vapor density at the surface
θ	dimensionless temperature difference
k	thermal conductivity of air

Superscript

$\hat{}$	non-dimensional parameter

Subscripts

atm	atmosphere property
st	static property
w	wall property

transfer analogy requires simple relations for the case of unequal Prandtl and Schmidt numbers.

Lewis [3] showed that the heat and mass transfer coefficients can be accurately related using an expression from universal velocity profiles in a turbulent boundary layer. Chen and Goldstein [4] used $n = 1/3$ to get an analogy factor with the Colburn relation $\frac{Nu}{Sh} = \left(\frac{Pr}{Sc}\right)^n$. Goldstein and Cho [5] reviewed the naphthalene sublimation technique and the heat/mass transfer analogy with a constant heat flux boundary condition. They recommended $n = 1/3$ for a laminar flow and $n = 0.14$ for wake regions in a Colburn type relation. Eckert et al. [6] compared experimental data with the heat/mass transfer analogy for laminar, turbulent, and three-dimensional flow. However, comparison across different studies is difficult due to the different geometric and experimental conditions. Yoo et al. [7] measured local and average mass transfer rates from a rectangular cylinder. They compared the mass transfer results with heat transfer results from a constant heat flux experiment. It was concluded that the discrepancy between heat and mass transfer results comes from the difference in the boundary condition, the effect of 'n' in the Colburn relation, conduction error in heat transfer and mechanical erosion in mass transfer. Apparently, no heat and mass transfer comparisons have been performed with equivalent boundary conditions on the same geometry for the evaluation of the analogy. With a turbulent boundary layer, the effect of different thermal boundary conditions may not be as important as in a laminar flow. However, the constant wall heat flux condition is not equivalent to the constant wall concentration condition and generally results in a higher heat transfer coefficient than the latter. Therefore, a constant wall temperature boundary condition is required to get accurate heat/mass transfer analogy factors when a comparison is to be made with a mass transfer study, with a constant wall concentration boundary condition.