International Journal of Heat and Mass Transfer 53 (2010) 5947–5961

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Analysis of heatlines for natural convection within porous trapezoidal enclosures: Effect of uniform and non-uniform heating of bottom wall

Tanmay Basak [a], S. Roy [b], Anjanna Matta [b], I. Pop [c,*]

[a] Department of Chemical Engineering, Indian Institute of Technology Madras, Chennai 600036, India
[b] Department of Mathematics, Indian Institute of Technology Madras, Chennai 600036, India
[c] Faculty of Mathematics, University of Cluj, R-3400 Cluj, CP 253, Romania

ARTICLE INFO

Article history:
Received 15 March 2010
Accepted 9 July 2010
Available online 7 September 2010

Keywords:
Penalty finite element method
Natural convection
Porous medium
Trapezoidal cavity
Various angles
Heatlines

ABSTRACT

In this paper, the numerical investigation of natural convection in a porous trapezoidal enclosures has been performed for uniformly or non-uniformly heated bottom wall. Penalty finite element analysis with bi-quadratic elements is used for solving the Navier–Stokes and energy balance equations. The numerical solutions are studied in terms of streamlines, isotherms, heatlines, local and average Nusselt numbers for a wide range of parameters $Da(10^{-5}–10^{-3})$, $Pr(0.015–1000)$ and $Ra(Ra = 10^3–10^6)$. At low Darcy number ($Da = 10^{-5}$), heat transfer is primarily due to conduction for all φ's as seen from the heatlines which are normal to the isotherms. As Da increases to 10^{-4}, convection is initiated and the thermal mixing has been observed at the central regime for all φ's. Distribution of heatlines illustrate that most of the heat transport for high Darcy number ($Da = 10^{-3}$) occurs from hot bottom wall to the top portion of cold side walls. It has been found that secondary circulations appear at the top corners of the cavity for $\varphi = 45°$, $60°$ and bottom corners of the cavity for $\varphi = 90°$ with $Pr = 0.015$, $Da = 10^{-3}$ and $Ra = 10^6$. The physical interpretation of local and average Nusselt numbers are illustrated using heatlines. For uniformly heated bottom wall with cold side walls, Nu_b values are maximum near the corners of bottom wall for all φ's irrespective to Da and Pr. In contrast, for non-uniformly heated bottom wall, the local Nusselt number (Nu_b) is found to be minimum near the corners of bottom wall and that is also found to be a sinusoidal variation with distance at high Da for all angles (φ). The Nu_s distribution is similar in both cases along the side wall except near the junction of hot and cold wall for all φ's. Overall, heat transfer analysis for bottom and side walls is presented in terms of average Nusselt numbers ($\overline{Nu_b}$, $\overline{Nu_s}$). The critical Ra numbers corresponding to the onset of convection are obtained at $Da = 10^{-3}$ for $Pr(0.015–1000)$. For $Da = 10^{-3}$, average Nusselt numbers ($\overline{Nu_b}$ and $\overline{Nu_s}$) increase exponentially beyond the critical Ra. Overall, the heat transfer rate is large for square cavity ($\varphi = 90°$) compared to other angles (φ) irrespective of heating patterns.

1. Introduction

Analysis of natural convection within fluid-saturated porous media is an important issue in engineering due to its wide applications in geophysics, heat exchangers, ground-coupled heat pumps, solar collectors, reactors, grain storage, cooling of computer systems and other electronic equipments etc. [1–5]. As indicated by the review studies of Martynenko and Khramtsov [6], Vafai [7], Ingham and Pop [8], natural convection phenomena in various enclosures filled with fluid-saturated porous media is also important in the engineering applications to help the design of efficient thermal systems. Numerical modeling may be employed to understand and analyze these systems. The advantage of numerical simulations is that the expensive experimental costs may be reduced.

A few recent investigations on natural convection in porous media are based on several applications [9–20]. Wang et al. [9] studied natural convection in an inclined enclosure filled with porous medium under magnetic field. Alvarez and Flick [10] predicted cooling kinetics of stack of food products. Study of non-Darcy natural convection of a non-Newtonian fluid in a porous cavity was carried out by Hadim [11]. Chen [12] investigated forced convection heat transfer in microchannel heat sinks. A few earlier researchers also studied various application of natural convection in porous media [13–17]. Natural convection in an enclosure filled with two layers of porous media are investigated numerically by Merrikh and Mohamad [18]. The focus of the work is on the validity of the Darcy model. The extended Darcy–Forchheimer model was also used to describe resistance to flow through the porous baffles by Miranda and Anand [19]. Lauriat and Prasad [20] also

* Corresponding author.
E-mail addresses: tanmay@iitm.ac.in (T. Basak), sjroy@iitm.ac.in (S. Roy), ma08m001@iitm.ac.in (A. Matta), popm.ioan@yahoo.co.uk (I. Pop).

0017-9310/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2010.07.026

Nomenclature

Da	Darcy number
g	acceleration due to gravity (m s^{-2})
k	thermal conductivity (W m^{-1} K^{-1})
H	height of the trapezoidal cavity (m)
Nu	local Nusselt number
p	pressure (Pa)
P	dimensionless pressure
Pr	Prandtl number
R	Residual of weak form
Ra	Rayleigh number
T	temperature (K)
T_h	temperature of hot bottom wall (K)
T_c	temperature of cold inclined wall (K)
u	x component of velocity
U	x component of dimensionless velocity
v	y component of velocity
V	y component of dimensionless velocity
X	dimensionless distance along x coordinate
Y	dimensionless distance along y coordinate

Greek symbols

α	thermal diffusivity (m^2 s^{-1})
β	volume expansion coefficient (K^{-1})
γ	penalty parameter
φ	angle of inclination
θ	dimensionless temperature
ν	kinematic viscosity (m^2 s^{-1})
ρ	density (kg m^{-3})
Φ	basis functions
ψ	streamfunction
ξ	horizontal coordinate in a unit square
η	vertical coordinate in a unit square

Subscripts

b	bottom wall
l	left wall
r	right wall
s	side wall

used this model to examine the natural convection in vertical porous layer and in a vertical enclosure filled with porous medium. A few investigations on natural convection within porous trapezoidal enclosures have been carried out by earlier researchers [21,22]. Baytas and Pop [21] have studied natural convection on trapezoidal porous enclosure where the top cylindrical surface is cooled and the bottom cylindrical surface is heated while the remaining two nonparallel plane sidewalls of the enclosure are adiabatic. Recently Varol et al. [22] studied buoyancy-driven flow and heat transfer in an inclined trapezoidal enclosure filled with a fluid-saturated porous medium heated and cooled from inclined side walls.

In general, above study on natural convection have been carried out with streamlines and isotherms. Note that, isotherms are generally used to illustrate the temperature distribution in a domain; however, isotherms may not be suitable to visualize the direction and intensity of heat transfer particularly in convection problems in which path of heat flux is not perpendicular to isotherm due to convection effect. Earlier studies show that distribution of thermal energy in a cavity with convective flow can be best studied with the help of heatlines. Heatline approach was first proposed by Kimura and Bejan [23] to visualize the convective heat transfer and the method has been extended to different applications by Morega and Bejan [24] and Dash [25]. A detailed review on applications of heatlines and masslines was also performed by Costa [26–30]. Heatline method for the visualization of natural convection in a complicated cavity studied by Dalal and Das [31]. Deng and Tang [32] analyzed heatlines for conjugate natural convection in a square cavity. Various applications using heatlines were studied by Mukhopadhyay et al. [33,34]. Further, Bejan [35] also reviewed earlier works to illustrate the use of heatline concept to visualize various physical situations. Recently, the heatline concept has been used by Kaluri et al. [36] for differentially heated porous square cavities with different combination of heating sources. Eventhough a number of numerical investigations has been carried out in this area, there is a lack on visualization of heat flow to analyze the optimal thermal mixing and temperature distribution within porous trapezoidal enclosures. It is also essential to study the heat transfer characteristics in complex geometries in order to obtain the optimal design of the container for various industrial applications and current study analyzes the energy flow using heatlines within porous trapezoidal enclosures.

The aim of the present study is to analyze heat flow due to natural convection within porous trapezoidal enclosures with hot bottom wall and cold side walls in presence of insulated top walls. The main objective is to examine thermal mixing within the cavity for various material processing applications. In the current study, we have used generalized non-Darcy model, neglecting the Forchheimer inertia term, to predict the flow in porous medium. This model based on volume averaging principles was developed by Vafai and Tien [37]. Numerical simulations were performed using Galerkin finite element method with penalty parameter to solve the nonlinear coupled partial differential equations for flow and temperature fields. The Galerkin method is further employed to solve the Poisson equation for streamfunctions and heatfunctions. Finite discontinuity exists at the junction of hot and cold walls leading to mathematical singularity. Solution of heatfunction for such type of situation demands implementation of exact boundary conditions and this problem is solved by considering the average temperature at the junction of hot and cold walls keeping the adjacent grid nodes at the respective wall temperature similar to earlier works [38]. Non-orthogonal grid generation has been done with iso-parametric mapping [38,39]. Numerical results are obtained to display the streamlines, isotherms and heatlines within porous trapezoidal enclosures and the heat transfer rate for bottom and side walls in terms of local and average Nusselt numbers.

2. Mathematical formulation and solution procedure

2.1. Velocity and temperature distributions

Let us consider the physical model of a trapezoidal cavity with the right wall inclined at an angle $\varphi = 45^\circ$, 60° and 90° with X axis as seen in Fig. 1(a)–(c), respectively. The boundary conditions of velocity are considered as no-slip on solid boundaries. Confined fluid within porous bed is considered as incompressible, Newtonian and the flow is assumed to be laminar. For the treatment of the buoyancy term in the momentum equation, Boussinesq approximation is employed for equation of the vertical component of velocity to account for the variations of density as a function of temperature and to couple in this way the temperature field to the flow field. It is assumed that the temperature of the fluid phase is