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Heatline approach for visualization of heat flow and efficient thermal mixing with discrete heat sources

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ABSTRACT

Heatline approach has been implemented to visualize heat transfer and to study efficient thermal mixing of laminar natural convective flow in a square cavity with distributed heat sources. Four different cases, depending upon the location of the heat sources on the walls of the cavity, are studied. Wide range of fluids ($Pr = 0.015-1000$) have been studied over a range of Rayleigh numbers ($Ra = 10^3-10^5$). Governing equations and Poisson-type of equations for streamfunction and heatfunction have been solved using penalty finite element method. Heatlines and streamlines are found to be adequate to visualize and understand heat energy distribution and thermal mixing occurring inside a cavity. Various qualitative and quantitative features on variations of local and average Nusselt numbers for test cases are adequately explained based on heatlines. The efficacy of distributed heating over conventional bottom heating for optimal thermal mixing is established via correlating with heatlines and cup-mixing temperature. The distributed heat sources are found to play major role for processing of molten materials and gaseous substances. Overall, it is shown that heatlines give suitable guidelines to assemble discrete heat sources and the heatline analysis on heat flow management with distributed heat sources is reported for the first time in this work.

1. Introduction

Natural convection involves transport of heat and mass by buoyancy induced circulation and this phenomena has many scientific and engineering applications. Studies of natural convection in recent applications include microchannels [1,2], cooling of computer chips [3], heat and mass transport in porous media [4–6], melting [7], efficient design of solar and microwave systems [8,9], sterilization of food materials [10,11], etc.

Natural convection in enclosures with discrete heat sources is the subject of prime importance as such configurations are found in various applications like cooling and packaging of electronic components, ventilation, material processing etc. [12–22]. In electronic equipments, chips and resistors constitute the discrete heat sources and the heat transfer from these heat sources affects the overall performance of the appliance. In materials processing applications, the discrete heat sources may be used to control the fluid flow and heat flow in the enclosure in order to improve the mixing and temperature distribution. In design of buildings with better indoor air quality, the heat from a discrete source such as fire place could be suitably located for comfortable thermal environment in a building. Also, excessive heat emitted by equipments like furnaces, boilers, offices automation devices, etc. may be exhausted by proper design of ventilation ports.

The presence of discrete heat sources in enclosures add complexities to the system as the localized heat sources lead to formation of intermittent boundary layers which interact with more than one global core flow. A detailed understanding of fluid flow and heat flow would be very useful in design of energy efficient systems. Considerable number of experimental and numerical studies have been reported on the topic of natural convection in discretely heated enclosures. Chu et al. [12] numerically investigated the effect of location of a single discrete heat source in a two-dimensional enclosure and found that the maximum heat transfer depends on location of the heat source and is a function of Grashof number. Keyhani et al. [13] have carried out experiments with an array of discrete heat sources in enclosures and concluded that heat transfer is significantly enhanced in the discrete cavity compared to the cavity with fully heated vertical wall. Coupled conduction and natural convective transport of water and a dielectric fluorocarbon liquid from an array of discrete heaters was investigated numerically and experimentally by Heindel et al. [14,15]. Aydin and Pop [16] numerically studied the effects of Rayleigh number, Prandtl number, length of a iso-

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Nomenclature

g	acceleration due to gravity, m s^{-2}
k	thermal conductivity, W m^{-1} K^{-1}
l'	dimensionless length of hot/cold section
L	side of the square cavity, m
N	total number of nodes
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
p	pressure, Pa
P	dimensionless pressure
Pr	Prandtl number
R	residual of weak form
Ra	Rayleigh number
T	temperature of the fluid, K
T_h	temperature of discrete heat sources, K
T_c	temperature of cold portions of the cavity, K
u	x component of velocity
U	x component of dimensionless velocity
v	y component of velocity
V	y component of dimensionless velocity
$\widehat{V}$	dimensionless velocity,
X	dimensionless distance along x coordinate
x	distance along x coordinate
Y	dimensionless distance along y coordinate
y	distance along y coordinate

Greek symbols

α	thermal diffusivity, m^2 s^{-1}
β	volume expansion coefficient, K^{-1}
γ	penalty parameter
Γ	boundary
θ, Θ	dimensionless temperature
ν	kinematic viscosity, m^2 s^{-1}
ρ	density, kg m^{-3}
Φ	basis functions
ψ	stream function
Π	heat function

Subscripts

i	residual number
k	node number
b	bottom wall
l	left wall
r	right wall
1,3	cold section
2	hot section
cup	cup-mixing
avg	spatial average

flux discrete heater, and material parameter on the momentum and heat transfer of a micropolar fluid in an enclosure with a discrete heater. Fitzgerald and Woods [17] made investigations on transient natural ventilation of a room with a distributed heat source while Livermore and Woods studied [18] the effect of distributed cooling in natural ventilation. Very few studies have been reported on the application of distributed heat sources for enhanced thermal processing of materials. Plumat [19] indicated that melting of glass can be enhanced by the use of discrete heat strips. Recently, the effect of the position of heat strip on flow currents, temperature distribution and thermal penetration in an industrial glass melting tank was investigated by Sarris et al. [20,21]. Melting of phase change materials by discrete heat sources was studied by Zhang et al. [22]. Ma et al. [23] have carried out studies on enhancement of oxygen transport in liquid metal by natural convection in a discretely heated enclosure.

Most of the experimental and numerical studies on enclosures with discrete heat sources in literature are presented and analyzed using streamlines and isotherms. Streamlines can adequately explain fluid flow [24], but isotherms are inadequate to study energy flow and its distribution as the isotherm contours indicate only temperature variations which are sufficient enough to study conduction heat transfer. In order to analyze convective heat transfer and heat flow distributions, there is a need for proper tool by which one can 'visualize' the energy flow. One such mathematical tool termed as 'heatlines' was developed by Kimura and Bejan [25]. The 'heatlines', analogous to streamlines, are trajectories of flow of heat energy and thus, they are useful to visualize the total energy flow in a two-dimensional domain. Heatlines are mathematically represented by heatfunctions and proper dimensionless form of 'heatfunctions' are closely related to Nusselt numbers. The heatline concept since then was applied and extended to various problems [26–54]. A comprehensive review of heatlines with various applications was reported by Costa [34].

However, to date, no attempt has been made to study thermal mixing by natural convection in enclosures with discrete heat sources using heatline approach. Thermal mixing has significance in processing of various materials like molten metals, chemical solutions, food materials, industrial fluids, etc., as it plays a important role in enhancing temperature distribution and product quality. Also, by enhanced thermal mixing with discrete heat sources, the need for external instruments for mixing, like stirrers, may not arise, which further results in energy savings. Visualization of heat flow by heatlines may provide a comprehensive understanding of thermal mixing due to discrete heat sources, which would immensely help in designing systems with high energy efficiency.

The aim of the present article is to implement the heatline concept to a square cavity with discrete heat sources and visualize the energy distribution with the help of 'Bejan's heatlines' along with fluid circulations and temperature distributions in the cavity. Also, to study the role of distributed heating over conventional bottom wall heating in enhancing the thermal mixing in the cavity through detailed heat flow analysis by heatlines and cup-mixing temperature. Four different cases, with various locations of discrete heat sources on the walls of the cavity are considered. In the current study, the non-linear coupled partial differential equations of flow and temperature fields have been solved using Galerkin finite element method. Finite element method is further employed to solve the Poisson equation for streamfunctions and heatfunctions. The advantage of this method is that the homogeneous Neumann boundary conditions are automatically built in the formulations. The heat transfer characteristics are studied by analyzing local and average Nusselt numbers as a function of Rayleigh number. Also, various qualitative features of local and average Nusselt numbers are adequately explained based on heatlines. Wide range of fluids, e.g., molten metals ($Pr = 0.015$), air ($Pr = 0.7$), aqueous chemical solutions, e.g., KOH ($Pr = 10$), olive/engine oil ($Pr = 1000$), which are of common interest for many scientific and engineering applications, are studied within $Ra = 10^3 - 10^5$.