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Laminar flow and convective heat transfer of non-Newtonian fluids in doubly connected ducts

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ABSTRACT

A hybrid numerical–analytical solution based on the Generalized Integral Transform Technique (GITT) is obtained for laminar heat and fluid flow of power-law non-Newtonian fluids inside doubly connected ducts. The mathematical formulation is constructed in the cylindrical coordinates system in such a way that the solid surfaces are described in terms of internal and external radii as functions of the angular coordinate, thus avoiding discontinuities in the boundary conditions. An annular doubly connected duct of arbitrary geometric configuration is considered for the analysis of the fully developed velocity field, as well as for the temperature field under thermally developing flow with boundary conditions of prescribed wall temperature. For illustration purposes, the case of eccentric annular ducts is more closely analyzed in order to demonstrate the ability of the GITT approach in dealing with such class of problems. Numerical results for the velocity field, the product of the Fanning friction factor-Reynolds number, temperature field and Nusselt numbers were produced for different values of the governing parameters, i.e., eccentricity, radii ratio and power-law indices. Such results were examined against previously reported ones, providing critical comparisons in order to illustrate the adequacy of the employed integral transform approach.

1. Introduction

Heat and fluid flow in doubly connected ducts is frequently found in a wide range of engineering and industrial applications in connection with heat exchange devices. Shah and London [1] pointed out a number of works that dealt with laminar flow inside annular passages and various other types of doubly connected geometries. Among them, elliptical ducts with central circular cores and eccentric annular ducts appear as the most relevant and frequently considered geometric configurations, mainly due their wide use in double-pipe type heat exchangers. On the other hand, the study of thermally developing flow in such configurations is important, mainly due to their usual employment in compact heat exchangers. In such type of thermal equipment, because of imperfections and tolerances in manufacturing, installation and operation, the eccentricity may or not be important, but there are a few applications where this effect is more pronounced and even promoted on purpose aimed at heat transfer enhancement. Oil and gas drilling wells, polymer and plastic extrusion process and nuclear reactors are some of the situations that reflect the importance of eccentricity. In addition, in dealing with purely viscous non-Newtonian fluids, heat and fluid flow analysis is commonly encountered in different industries. Chemical, food processing and pharmaceutical applications are just a few typical situations in which the power-law model can adequately describe the rheology of the working fluids.

The compilations in [1–3] provide extensive information on fully developed laminar flow of Newtonian fluids in ducts with doubly connected cross-sections. A variety of different methods has been employed in the literature to obtain solutions for the governing partial differential equations and associated boundary conditions, most based on discrete numerical techniques. Among the more analytically oriented contributions, one may cite the pioneering works of Piercy et al. [4], Sastry [5,6], Shivakumar [7] that employed conformal mapping methods, and Topakoglu and Arnas [8], which used an elliptical coordinates system to analyze the flow in confocal elliptical ducts. Solutions in closed-form were obtained by these authors for the velocity field and related flow characteristics. Shivakumar [7] also analyzed the flow in elliptical ducts with central circular cores through conformal mapping. Attention has also been devoted to the analysis of flow and heat transfer in eccentric annular ducts. The literature review brings up some of the earlier studies on such class of ducts, which are attributed to Piercy

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Nomenclature

A_{ij}	coefficient defined by Eq. (20)
A_t	dimensionless parameter related to the cross-sectional area of the annular passage
B_i	coefficient defined by Eq. (21)
C_{ij}	coefficient defined by Eq. (22)
c_p	specific heat
D_h	hydraulic diameter
D_{ij}	coefficient defined by Eq. (23)
f	Fanning friction factor
h_{iw}, h_{ow}	local heat transfer coefficient at the inner and outer walls, respectively
k	thermal conductivity
K	consistency index of the fluid
L_i	normalization integral for the velocity field
L_1, L_2	radial characteristic length of the inner and outer surfaces, respectively
M_i	normalization integral for the temperature field in the R-direction
n	power-law index
N_ℓ	normalization integral for the temperature field in the θ-direction
NT, NT^*	truncations orders for the temperature expansion
Nu_{iw}, Nu_{ow}	local Nusselt number at the inner and outer walls, respectively
NV	truncation order for the velocity expansion
Pe	Peclet number
$P_{ij\ell m}$	coefficient defined by Eq. (66)
Pr	Prandtl number
$Q_{ij\ell m}$	coefficient defined by Eq. (67)
r, R	radial coordinates, dimensional and dimensionless, respectively
Re	Reynolds number
r_i, r_o	inner and outer radii of the eccentric annular duct, respectively
R_n	normalized radial coordinate
$\tilde{\bar{f}}_{i\ell}$	transformed inlet temperature
r_1, R_1	functions that describe the inner surface, dimensional and dimensionless, respectively
r_2, R_2	functions that describe the outer surface, dimensional and dimensionless, respectively
T^*, T	temperature distributions, dimensional and dimensionless, respectively
T_e^*	inlet temperature
T_{iw}^*, T_{ow}^*	prescribed temperatures at the inner and outer walls, respectively
T_m	dimensionless average temperature
$\bar{T}_{i\ell}$	transformed potential for the temperature field
$v_{z,m}^*, V_{z,m}$	average flow velocity, dimensional and dimensionless, respectively
v_z^*, V_z	velocity distributions, dimensional and dimensionless, respectively
$\bar{V}_{z,i}$	transformed potential for the velocity field
z, Z	axial coordinates, dimensional and dimensionless, respectively

Greek letters

α	fluid thermal diffusivity
β_i	eigenvalues defined by Eq. (11)
γ	radii ratio
Γ_i	eigenfunctions defined by Eq. (10)
ε^*, ε	eccentricities, dimensional and dimensionless, respectively
θ	angular coordinate
λ_i	eigenvalues defined by Eq. (56)
μ_ℓ	eigenvalues defined by Eq. (57)
ρ	density
Ψ_i	eigenfunctions defined by Eqs. (53) and (54)
Ω_ℓ	eigenfunctions defined by Eq. (55)

Subscripts and superscripts

i, j, k, ℓ, m	expansions indices
iw, ow	relative to the inner and outer wall, respectively
m	relative to average quantities
$-$, $\tilde{-}$	integral transformed quantities

et al. [4], Stevenson [9], Snyder and Goldstein [10] and Jonsson and Sparrow [11] that concentrated their analyses on fluid flow only, while Cheng and Hwang [12], Trombetta [13] and Suzuki et al. [14] analyzed the heat transfer problem under different sets of boundary conditions. More recently, heat transfer in doubly connected ducts have regained interest in the works of Manglik and Fang [15], Fang et al. [16], Manglik and Fang [17], Escudier et al. [18], Coelho and Pinho [19], and Pinho and Coelho [20], in which the effects of eccentricity, duct rotation and viscous dissipation were investigated for the flow and heat transfer of Newtonian and non-Newtonian fluids as well. In addition, the recent work by Escudier et al. [18] offers the literature review on flow and heat transfer in eccentric annular ducts involving Newtonian and non-Newtonian fluids.

Within a wider framework, a hybrid analytical–numerical approach has been progressively advanced for the solution of convection–diffusion-type problems defined within irregular domains [21], and applied to the analysis of fully developed laminar flow within ducts of various shapes, such as trapezoidal, triangular, and hexagonal ducts [21–24], by extending the ideas on the Generalized Integral Transform Technique (GITT), as reviewed by Cotta [24,25] and Cotta and Mikhailov [26]. Laminar flow and heat transfer of Newtonian and non-Newtonian fluids inside irregular ducts of different geometric configurations was also treated in [27–33], again through extension of the GITT approach, yielding accurate numerical results for quantities of practical interest such as the Fanning friction factor and Nusselt numbers, within a wide range of the governing parameters.

Following this same line of research, the purpose of the present study is to solve the momentum and energy equations for laminar fully developed flow and thermally developing flow of non-Newtonian power-law fluids in doubly connected ducts by employing the GITT approach, and to provide reference results for the velocity field, the product of the Reynolds number-Fanning friction factor, temperature field and Nusselt numbers, for different values of the governing parameters according to the specific cross-section under consideration and thermal boundary conditions either at the inner or outer duct walls. The cylindrical coordinates system is used in the mathematical formulation, so that the solid surfaces are described in the form of internal and external radii as functions of the angular coordinate, and thus avoiding more involved formulations in other coordinates systems and cumbersome domain transformation approaches. The analysis is first accomplished in an arbitrary doubly connected duct in order to find a general solution for the velocity field and then for the temperature field. To illustrate the present approach, the case of eccentric annular ducts is considered, so as to demonstrate the ability of the GITT methodology in handling such class of problems. The thermal field is more