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Fast transient thermal analysis of Fourier and non-Fourier heat conduction

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Abstract

In this paper, asymptotic waveform evaluation (AWE) has been successfully used for fast transient characterization of Fourier and non-Fourier heat conduction. The Fourier and non-Fourier equations are reduced to a system of linear differential equations, respectively, using finite element method and then solved with AWE. Besides providing equivalent accuracy in its solution, it is also shown that AWE is at least three orders faster in term of computational time as compared to conventional iterative solvers. Its accuracy is also independent of the time step used and it has the capability of providing local transient solution. However, the moment matching process in AWE is inherently ill-conditioned and thus may yield unstable response even for stable system. This numerical instability is addressed and two stability schemes are also successfully implemented to yield stable and accurate solutions from AWE. The limitation of AWE is also discussed.

Keywords: Asymptotic waveform evaluation (AWE); Fourier; Non-Fourier; Transient; Conduction

1. Introduction

Asymptotic waveform evaluation (AWE), which has been used for fast transient circuit simulation, is based on the concept of approximating the original system with a reduced order system. The inspiration of AWE came from Rubinstein et al. [1], where RC-tree networks were estimated using efficient Elmore delay approach. However, these estimates were not always accurate. A second breakthrough came from the work of McCormick [2], in which he has used the interconnect circuit moments to form a lower order circuit models to predict transient responses accurately. The efforts of these authors lead to the formalization and generalization of AWE algorithms [3,4].

For more than a decade, extensive works on AWE has been carried out. AWE has been successfully applied for fast transient circuit simulation [5–7]. AWE also has a lot of successes in electromagnetic simulations. However, there are only two papers available on the application of AWE in transient thermal simulation. Da et al. [8] have published the first paper on thermal analysis of PCB using AWE scheme, but the details of incorporating the initial conditions were not addressed. Then, Ooi et al. [9] has successfully extended the AWE algorithm to incorporate the initial conditions. They created a generalized formulation using the concept of zero state response and zero input response, which is used in control system. However, Ooi et al. [9] did not address the inherent numerical instability of AWE, which may yield incorrect solutions. Both papers

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Nomenclature

k	residue	Z_q	dimensionless phase lag for heat flux
M	moment	β	dimensionless time
p	pole	δ	dimensionless distance x
t	time (s)	ε	dimensionless distance y
T	temperature (°C)	θ	dimensionless temperature
Z_T	dimensionless phase lag for temperature gradient		

also only focused on solving Fourier heat conduction equation with AWE.

On the other hand, finite element method (FEM) has been extensively used to solve thermal problems because it is capable to account for complicated three-dimensional geometry. Besides that, the governing equations for Fourier and non-Fourier heat conduction are also parabolic and hyperbolic in nature, respectively, and they are difficult to be solved analytically. Using FEM, the transient heat conduction equation (partial differential equation) is reduced to a set of linear differential equations through the process of discretization. This set of differential equations can then be solved in time domain to obtain its transient solution.

Usually, this set of equations is solved using conventional iterative solvers such as Crank–Nicolson, Runge–Kutta and the famous Newmark algorithm. These conventional numerical solvers require the whole set of equations to be solved at each increment of time step, even though only the solution at a particular node is of interest. Solving this large set of equations is very time consuming, especially when the time step required is also very small in order to yield accurate solutions.

In contrast, AWE is actually approximating the original system with reduced order system and thus, it is a few orders faster than conventional iterative solvers in term of computational time. It is also independent of time step because it produces the transient solutions in a form of equation, rather than numerical solutions at every increment of time step. AWE is also capable of producing local solution because it can obtain the solution for each node independently and thus further reducing the amount of computational time. However, the drawback of AWE is that the moment matching process in AWE is inherently ill-conditioned and thus may produce unstable response even for stable system [10]. Higher order approximation will lead to a more accurate solution but not always guarantee a stable solution.

In this paper, FEM is coupled with AWE to efficiently solve the transient Fourier and non-Fourier heat conduction equations. FEM is used to reduce the Fourier (parabolic) and non-Fourier (hyperbolic) equations to a set of first and second order linear differential equations, respectively. AWE is then used to obtain the transient solutions instead of using conventional iterative solvers. The inherent instability of AWE is also addressed and two stability schemes are also introduced to yield accurate and yet stable solution even using higher order approximation.

2. Mathematical model for Fourier and non-Fourier heat conduction

Classical Fourier's law is based on diffusion model with assumption of infinite thermal wave propagation speed, which leads to simultaneous development of heat flux and temperature gradient. Classical Fourier's law also assumes that instantaneous local thermal equilibrium occurs between electrons and phonons. In other words, classical Fourier's law dictates that the thermal effect is felt instantaneously throughout the system if the surface of a material is heated. The governing equation for non-dimensionalized two-dimensional Fourier heat conduction is a parabolic equation as shown by Eq. (1).

$$\frac{\partial^2\theta}{\partial\delta^2}+\frac{\partial^2\theta}{\partial\varepsilon^2}=\frac{\partial\theta}{\partial\beta} \tag{1}$$

where θ is the dimensionless temperature and β is the dimensionless time. The dimensionless distance x and y are represented by δ and ε, respectively.

After discretizating Eq. (1) with Galerkin's weighted residual method, a set of first order linear differential equations is obtained as given by Eq. (2). The detailed formulations of Eq. (2) can be obtained from Logan [11].

$$C\dot{\theta}+K\theta=f \tag{2}$$

where C is known as the capacitive matrix, while K is the conductivity matrix. f represents the load vector, which can be time-dependent or time-independent.

Classical Fourier law is sufficient for most heat conduction phenomena, but it is inadequate to describe rapid heating response, such as VLSI interconnection heating. Thus, many non-Fourier heat conduction equations are proposed by many researchers to account for the finite thermal wave propagation speed and/or finite relaxation time to establish local thermal equilibrium between electrons and phonons. The non-Fourier model discussed in this paper is a two-phase lag model proposed by Tzou