International Journal of Heat and Mass Transfer 53 (2010) 2643–2654

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Inverse estimation of surface heating condition in a three-dimensional object using conjugate gradient method

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ARTICLE INFO

Article history:
Received 30 October 2009
Received in revised form 22 January 2010
Accepted 22 January 2010

Keywords:
Inverse heat conduction
Laser
Gaussian profile
Conjugate gradient method

ABSTRACT

Temperature and heat flux on inaccessible surfaces can be estimated by solving an inverse heat conduction problem (IHCP) based on the measured temperature and/or heat flux on accessible surfaces. In this study, the heat flux and temperature on the front (heated) surface of a three-dimensional (3D) object is recovered using the conjugate gradient method (CGM) with temperature and heat flux measured on back surface (opposite to the heated surface). The thermal properties of the 3D object are considered to be temperature-dependent. The simulated measurement data, i.e., the temperature and heat flux on the back surface, are obtained by numerically solving a direct problem where the front surface of the object is subjected to high intensity periodic laser heat flux with a Gaussian profile. The robustness of the formulated 3D IHCP algorithm is tested for two materials. The effects of the uncertainties in thermophysical properties on the inverse solutions are also examined. Efforts are made to reduce the total number of heat flux sensors on the back surface required to recover the front-surface heating condition.

1. Introduction

High-Energy Laser (HEL) weapons can remotely deliver high-power laser at the speed of light onto a military target. It is critical to know the transient of temperature in the target in order to accurately assess the resulting thermomechanical response. However, the conventional temperature sensors cannot be used to directly measure the surface temperature since the sensors can be easily destroyed or interfere with the laser beam. Similar problems can be found during reentry of a space vehicle into the atmosphere as well as in high-power laser manufacturing processes [1]. For these circumstances, however, the heated surface temperature can be determined indirectly by solving an inverse heat conduction problem (IHCP) [2–4] based on the transient temperature and/or heat flux measured on the back surface.

To formulate the IHCP, either temperature or heat flux at some locations should be measured to provide information for solving the ill-posed problem. Between them, temperature is often preferred because it can be measured with less uncertainty compared to the heat flux [5–8]. Recent studies, however, have shown that using the measured heat flux as additional information in an IHCP can reduce the proneness to the inherent instability of the ill-posed problem [9,10].

Although the IHCPs have been extensively studied for different applications in the past decades (e.g., [11–19]), little work has been done for the inverse numerical algorithm using heat flux measurement data in the objective functional. Furthermore, in HEL weapon applications, the laser energy may be delivered to the surface in a periodic way because of the target-spinning or atmosphere variations. Since the formulation of the IHCP is quite subjective, it is necessary to determine which formulation is more appropriate for applications with a periodic heat flux that may pose extra difficulties in the solution of the inverse problems.

Recently, the authors proposed a stable 1D IHCP formulation to reconstruct the front-surface heating condition with back-surface measurement data [20]. After an optimal investigation on the choice of the boundary condition and objective function variable, it was demonstrated that the most accurate solution can be obtained by choosing the front-surface heat flux as an unknown function, using the temperature measurement data as the boundary condition at the back surface, and employing the heat flux measurement data in the objective function. In Ref. [20] thermophysical properties were assumed to be constant. In reality, those properties could vary with temperature during a high-power laser interaction.

The 1D IHCP model can be applied to the situation that the flat-top laser beam diameter is much larger than the thickness of the target. For the case that the laser beam profile is Gaussian and/or the laser diameter is comparable to the thickness of the target, the temperature distribution in the target is 3D. The objective of this paper is to develop a 3D IHCP formulation that can accurately recover the front surface temperature based on measured temperature and heat flux on the back surface for a target subjected to a

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doi:10.1016/j.ijheatmasstransfer.2010.02.048

Nomenclature

C	dimensionless volume specific heat
C_*	volume specific heat (J/m^3 K)
$d^k(y,z,t)$	dimensionless direction of descent at iteration k, which is sometimes expressed in vector form $\mathbf{d}^k$
f	frequency of periodic laser heat flux on front surface (Hz)
h	dimensionless convection heat transfer coefficient
h_*	convection heat transfer coefficient (W/m^2 K)
i_m	total number of heat flux measurements
k	dimensionless thermal conductivity
k_*	thermal conductivity (W/m K)
l_c	characteristic length (m)
L	dimensionless object length in x direction
L_*	object length in x direction (m)
M	dimensionless object length in y direction
M_*	object length in y direction (m)
N	dimensionless object length in z direction
N_*	object length in z direction (m)
q	dimensionless heat flux
$\mathbf{q}$	vector form of dimensionless heat flux
q_c	characteristic heat flux (W/m^2)
q_{laser}	dimensionless periodic laser heat flux on front surface
q^*_{laser}	periodic laser heat flux on front surface (W/m^2)
q_{max}	dimensionless maximum heat flux at the laser Gaussian beam center
q^*_{max}	maximum heat flux at the laser Gaussian beam center (W/m^2)
$q_1(y,z,t)$	dimensionless observed heat flux on front surface which is sometimes expressed in vector form $\mathbf{q}_1$
$q^*_1(y,z,t)$	observed heat flux on front surface which is sometimes expressed in vector form $\mathbf{q}_1$ (W/m^2)
$q[L,y,z,t;\mathbf{q}_1]$	dimensionless observed heat flux on back surface
$\Delta q[L,y,z,t;\mathbf{d}^k]$	dimensionless heat flux variation, which is sometimes simplified as $\Delta q(\mathbf{d}^k)$, when the surface heat flux is perturbation is $\Delta q_1(y,z,t) = d^k(y,z,t)$
r	radius measured from laser spot center (m)
S	dimensionless objective function
$\nabla S[\mathbf{q}^k_1]$	dimensionless gradient direction of objective functional at iteration k
$\Delta S[\mathbf{q}^k_1]$	dimensionless objective function variation
t	dimensionless time
t_*	time (s)
t_c	characteristic time (s)
t_f	dimensionless final time
t^*_f	final time (s)
Δt	time step
Δt_*	time step (s)
T	dimensionless temperature
T_c	characteristic temperature (K)
T_0	dimensionless initial temperature
T^*_0	initial temperature (K)
T_∞	dimensionless ambient temperature
T^*_∞	ambient temperature (K)
$T_1(y,z,t)$	dimensionless front surface temperature
$T^*_1(y,z,t)$	front surface temperature (K)
$\Delta T[L,y,z,t;\mathbf{d}^k]$	dimensionless temperature variation, which is sometimes simplified as ΔT, when the surface heat flux is perturbation is $\Delta q_1(y,z,t) = d^k(y,z,t)$
w	dimensionless $1/e$ radius of Gaussian laser beam
w_*	$1/e$ radius of Gaussian laser beam (m)
x, y, z	dimensionless spatial coordinate variables
x_*, y_*, z_*	spatial coordinate variables (m)
$Y(y,z,t)$	dimensionless measurement data (temperature or net heat flux) with errors on back surface obtained by numerical simulations
$Y_{exact}(y,z,t)$	dimensionless measurement data (temperature or net heat flux) without errors on back surface obtained by numerical simulations
$Y_{qL}(y,z,t)$	dimensionless measurement net heat flux on back surface
$Y^*_{qL}(y,z,t)$	measurement net heat flux on back surface (W/m^2)
$Y_{TL}(y,z,t)$	dimensionless measurement temperature on the back surface
$Y^*_{TL}(y,z,t)$	measurement temperature on back surface (K)

Greek symbols

α	surface absorptivity
β^k	dimensionless search step size at iteration level k
χ	dimensionless tolerance used to stop the CGM iteration procedure
δ	Dirac delta function
ε	surface emissivity
ϕ	dimensionless standard deviation of heat flux or temperature measurements
ϕ_*	standard deviation of heat flux (W/m^2) or temperature (K) measurements
γ^k	dimensionless conjugate coefficient at iteration level k
$\lambda(x,y,z,t)$	dimensionless Lagrange multiplier
σ	a dimensionless quantity related to Stefan–Boltzmann constant, defined by Eq. (1)
σ_*	Stefan–Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W/m^2 K^4
ω	a dimensionless random variable having a normal distribution with zero mean and unitary standard deviation
ξ	dimensionless perturbed variable

Superscripts

*	real physical quantities with dimensions
k	iteration level

Subscripts

0	initial
f	final
q	heat flux
T	temperature

periodic heat flux on the front surface and with temperature-dependent thermophysical properties.

2. Model description

To illustrate the methodology of the inverse heat transfer algorithm employed in this study, a three-dimensional object shown in Fig. 1 is considered. Initially, the object is under a uniform temperature T^*_0 and then is subjected to a high intensity, Gaussian laser beam q^*_{laser} on the front surface from $t_* = 0^+$. The purpose of this study is to demonstrate the effectiveness and accuracy of the proposed IHCP formulation in reconstructing the observed heat flux $q^*_1(y,z,t)$ and temperature $T^*_1(y,z,t)$ on the front surface of a 3D target with temperature-dependent thermophysical properties, based on the measured temperature and heat flux on back surface. Due to the fact that temperature measurement contains much less errors compared to the heat flux measurement [5–8], the back surface temperature $Y^*_{TL}(y,z,t)$ is used as the boundary condition while the back-surface heat flux $Y^*_{qL}(y,z,t)$ is adopted in the objective function.