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An RBF–MFS model for analysing thermal behaviour of skin tissues

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ABSTRACT

A meshless model based on radial basis function and method of fundamental solution (RBF–MFS) is developed to investigate bioheat transfer problems. First, A time-stepping θ-method is used in handling the time variable in the Pennes bioheat equation. Then, the particular solution is approximated by a linear combination of radial basis functions, and the homogeneous solution is approximately determined by the method of fundamental solution. The multi-subdomain RBF–MFS technique is implemented for analysing problems containing different materials and/or multi-connected regions. The efficiency of the proposed method is assessed by several examples including normal tissue, tissue with tumor and burned tissue.

1. Introduction

Thermal methods of temperature measurement at the skin surface, which require solutions of generalized bioheat equations under various specific internal and boundary conditions to simulate the real case, are becoming recognized as more attractive than other non-invasive thermometry like MRI, microwave and ultrasound [1] because they are more economic and safer[2]. Research on the prediction of living tissue temperature has developed continuously since the Pennes equation was proposed in 1948 [3]. Numerical methods used to solve the Pennes equation have included the finite difference method (FDM) [4–6], finite element method (FEM) [7–11], boundary element method (BEM) [12,13], dual reciprocity boundary element method (DRMBEM) [2,14] and Monte Carlo method (MCM) [15,16]. In addition, the Trefftz FEM [17,18] and meshless method [19] have also been successfully used to solve transient heat conduction problems. Among the above methods, the major drawback of FDM appears to be in its inability to handle effectively the solution of problems over arbitrarily shaped complex geometries because of interpolation difficulties between the boundaries and the interior points in order to develop finite difference expressions for nodes next to the boundaries. FEM is widely used because it can manage complex shapes well, but its main disadvantage is that it requires domain discretization which is time-consuming. BEM involves discretization of the boundary only, which is an important advantage over FEM, but it has difficulty dealing with transient or non-homogeneous problems which still need domain discretization. Fortunately, DRMBEM can overcome this drawback by combining radial basis functions and conventional BEM to transform domain integrals to the boundary integral. An alternative numerical method is MCM, which differs from the classical numerical methods listed above because it is based on a random process approach and depends weakly on the dimension of the problem, providing an alternative way to deal with multidimensional problems.

Unlike the above approaches, in this paper a meshless RBF–MFS model is developed by combining radial basis function (RBF) approaches and the method of fundamental solutions (MFS) [19,20], to predict the temperature distribution in skin tissue. Firstly, the time dependence in the Pennes equation is removed by a time-stepping process and then the system is replaced by a set of inhomogeneous modified Helmholtz equations. Then, RBF approximation and the method of fundamental solution are employed to construct the particular and the homogeneous solution of the modified Helmholtz equation, respectively. The muti-subdomain method is employed to extend this model to problems with two inhomogeneous domains, such as skin with tumor, which can induce different frequencies in the modified Helmholtz equation system. The paper is organized into the following sections. In Section 2 a detailed numerical implementation is described and some important points of the proposed model are discussed. Section 3 provides some numerical examples which cover typical situations in thermal diagnostics, to demonstrate the effectiveness of the proposed method. Finally, Section 4 presents some conclusions from the presented analysis.

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Nomenclature

Alphabetical symbols

c	specific heat of tissue (J/kg/°C)
c_b	specific heat of blood (J/kg/°C)
h_∞	conventional coefficient (W/m^2/°C)
k	thermal conductivity of tissue (W/m/°C)
M	number of collocation points on the boundary
N_I	number of interpolation points in the domain
N_S	number of source points outside the domain
Q_m	metabolic heat of tissue (W/m^3)
Q_r	spatial heating (W/m^3)
Q_t	sum of metabolic heat and spatial heating (W/m^3)
q	normal heat flux (W/m^2)
t	time (s)
u	temperature (°C)
u_0	initial temperature (°C)
u_a	artery temperature (°C)
u_e	environmental temperature (°C)
u_w	temperature contact with probe (°C)

Greek symbols

λ	frequency of the modified Helmholtz equation
α	parameter defined in Eq. (19)
β	parameter defined in Eq. (28)
γ	parameter defined in Eq. (31)
τ	time step size
ρ	density of tissue (kg/m^3)
ρ_b	density of blood (kg/m^3)
ω_b	blood perfusion (m^3/s/m^3 tissue)
θ	temporal weighting in time-stepping method

Superscripts

1	subdomain Ω_1
2	subdomain Ω_2
n	time level n
$n+1$	time level $n+1$

Subscript

I	interface boundary between tissue domain and tumor domain.

2. Numerical method and algorithms

2.1. Pennes bioheat mathematical model

The well-known Pennes equation, which involves the effects of blood perfusion and metabolic heat generation, is used to simulate the thermal behaviour of biological tissue [3]:

$$\rho c\frac{\partial u(\mathbf{x},t)}{\partial t}=\nabla\cdot[k\nabla u(\mathbf{x},t)]+\omega_b\rho_b c_b[u_a-u(\mathbf{x},t)]+Q_m+Q_r(\mathbf{x},t) \tag{1}$$

where ρ, c, k are the density, specific heat, and thermal conductivity of the tissue, respectively; ω_b, ρ_b, c_b represent blood perfusion, density and specific heat of blood, respectively. u_a is the arterial temperature which is treated as constant, $u(x,\ t)$ is the tissue temperature; Q_m is the metabolic heat generation and $Q_r(x,\ t)$ is the heat source due to spatial heating. For convenience, a new symbol $Q_t(\mathbf{x},\ t) = Q_r(\mathbf{x},\ t) + Q_m$ including metabolic heat and special heating is introduced.

From the Pennes' equation, it can be seen that the first term on the right side represents conduction of heat in the tissue, caused by the temperature gradient. The second term describes the heat transport between the tissue and microcirculatory blood perfusion. The third term on the right depicts internal heat generation due to metabolism and the last term is spatial heating caused by external heat sources.

Practically, a rectangular area is often used in two-dimensional bioheat transfer problem (see [2,16,21]). A schematic of the 2D calculation geometry is depicted in Fig. 1

Without losing generality, the following boundary conditions and initial condition are applied to the four boundaries to make the system complete:

- Dirichlet/necessary condition

$$u(\mathbf{x},t)=\bar{u}(\mathbf{x},t)\quad\in\Gamma_u \tag{2}$$

- Newman/nature condition

$$q(\mathbf{x},t)=\bar{q}(\mathbf{x},t)\quad\in\Gamma_q \tag{3}$$

- convective condition

$$q(\mathbf{x},t)=h_e[u(\mathbf{x},t)-u_e]\quad\in\Gamma_c \tag{4}$$

- initial condition

$$u(\mathbf{x},0)=u_0\quad\in\Omega \tag{5}$$

where q represents the boundary normal heat flux defined as $q=-k\frac{\partial u}{\partial n}$ and n is the unit outward normal to the boundary Γ of the domain of interest Ω.

For convenience, boundary conditions (2)–(4) are expressed in a general form as

$$B_1u(\mathbf{x},t)+B_2q(\mathbf{x},t)=B_3(\mathbf{x},t) \tag{6}$$

where B_1, B_2, and B_3 are known coefficients and can be written respectively as

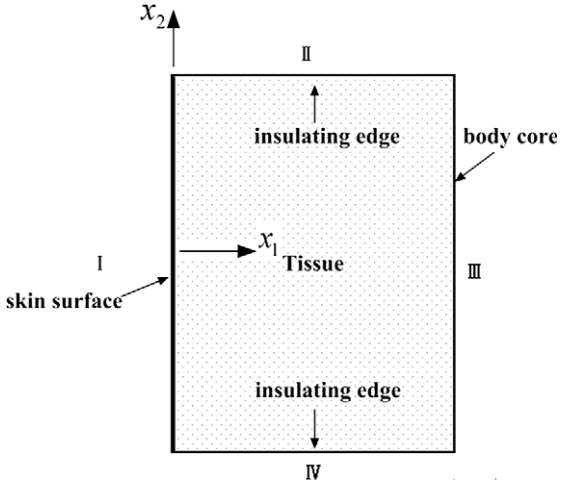

Fig. 1. Schematic diagram of computational area.