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Notable physical anomalies manifested in non-Fourier heat conduction under the dual-phase-lag model

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Abstract

In this study, a number of notable physical anomalies concerning non-Fourier heat conduction under the dual-phase-lag (DPL) model are observed and investigated. It is found that, during the transient heat transfer process, the over-diffusion mode predicts a "hyper-active" to "under-active" transition in thermal behavior. The main cause behind it lies in the time-varying effect of τ_T (the phase lag of the temperature gradient) on the thermal response. Also, change of polarity in reflected thermal waves can be observed when a constant-temperature boundary is involved, which hints that a heating process may be followed by a spontaneous cooling effect. A fairly strong connection is present between the τ_T-induced dispersive effect and an unusual thermal accumulation phenomenon in an on–off periodic heating process. Furthermore, a paradox involving a moving medium is detected in the DPL model, which can be solved by replacing the temporal partial derivatives in the DPL equation with the material derivatives. During the process of analysis, a high-order characteristics-based TVD scheme is relied on to provide accurate and reliable numerical simulations to the DPL heat conduction equation under various initial-boundary conditions.

Keywords: Non-Fourier heat conduction; Dual-phase-lag model; Total-variation-diminishing scheme; Physical anomalies

1. Introduction

After witnessing a myriad of substantive and comprehensive breakthroughs in the science of heat transfer in the past century or so, Fourier's law of heat conduction remains as one of the few empirical theories that are solely based on experimental observations and, at the same time, enjoying a large variety of applications in daily engineering practice nevertheless. Despite the fact that, Fourier's law implies that thermal signal travels at an infinite velocity when combined with energy conservation equation, it is fair to say that the classical heat conduction model yields excellent descriptions of various thermal transportation behaviors in a single-phase medium under most circumstances. However, when it comes to high-rate heating process (duration time on the order of femtoseconds) [1] or helium cryogenic engineering (near absolute zero) [2], the wave nature of heat conduction which is displayed in these situations, such as sharp wave front and finite velocity of thermal propagation, dismisses Fourier's law completely incapable—if not useless—to provide reasonable and decent predictions that capture these non-Fourier effects.

To alleviate this physically unrealistic notion of instantaneous energy diffusion, Cattaneo [3] and Vernotte [4] postulated a modified Fourier's law, commonly known as the thermal wave model, which in essence hypothesizes that there exists a time delay between the temperature gradient imposed and the heat flux to generate. This newly-developed hyperbolic heat conduction model, to some extent, fills the holes in the theoretical foundation of the classical diffusion theory and shows a great amount of promise of superseding it when extreme situations are involved (ultra-short period of time, small domain, exceedingly sharp temperature gradient or very low temperature) [5–9]. However, discernible discrepancies between theoretical predictions

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Nomenclature

c_p	specific heat capacity at constant pressure
$\mathbf{E}$	vector defined in Eq. (10)
$\mathbf{F}$	vector defined in Eq. (10)
$\mathbf{G}$	vector defined in Eqs. (12c) and (44)
k	thermal conductivity
L	characteristic length
n	number of period in Eq. (25)
P	duration of period in Eq. (25)
q	heat flux
$\mathbf{q}$	heat flux vector
Q	dimensionless heat flux
$\mathbf{r}$	spatial vector
$\mathbf{S}$	vector defined in Eq. (10)
t	time variable
T	temperature
T_r	reference temperature
u	velocity
$\mathbf{u}$	velocity vector
U	dimensionless temperature
$\mathbf{v}$	relative velocity vector of a moving frame to a medium in motion
$\mathbf{V}$	velocity vector of a moving frame
$\mathbf{W}$	vector defined in Eqs. (12a) and (42)
x	spatial variable
Z	dimensionless phase lag

Greek symbols

α	thermal diffusivity
β	dimensionless time
θ	dimensionless temperature in a moving frame
Λ	dimensionless velocity of a moving medium
λ_j	eigenvalue, $j = 1,2$
ξ	dimensionless spatial variable
P	density in a moving frame
ρ	density
τ	phase lag
χ	dimensionless spatial variable in a moving frame
$\boldsymbol{\chi}$	spatial vector in a moving frame
Ψ	dimensionless heat flux in a moving frame
$\boldsymbol{\Psi}$	heat flux vector in a moving frame
Ω	temperature in a moving frame

Subscripts

q	heat flux
T	temperature gradient

and experimental results [10], along with lacking in firm physical basis to support [11], leave the validity of the thermal wave model open to debate. As opposed to the "crisis" encountered in the application of the hyperbolic heat conduction model, the two-step model pioneered by Anisimov et al. [12] and the pure phonon field in dielectric crystals successfully obtained by Guyer and Krumhansl [13] helped shed light on the mechanism of the non-equilibrium thermodynamic transition and the energy exchange on the microscopic level. To incorporate these microscopic effects into a macroscopic description (that most practicing engineers are more acquainted with), Tzou [14] proposed the widely-celebrated dual-phase-lag (DPL) model. This universal model is claimed to be able to smoothly bridge the gap between the microscopic and the macroscopic approach, covering a wide range of heat transfer models (reducing to the classical diffusion model, the controversial thermal wave model, the two-step phonon–electron interaction model for metals, and the pure phonon scattering model for insulators, semiconductors and dielectric films, simply by adjusting the value of τ_q and τ_T, the two primary time relaxation parameters in the governing equation).

Recently the DPL heat conduction model has stimulated considerable interest in the heat transfer community, by offering alternative interpretations and new perspectives to a large body of non-Fourier thermal behaviors in energy transportation process under special considerations, such as heat conduction in biological materials [15], heat transport in amorphous media [16] and layered-film heating in superconductors, fins and reactor walls [17]. Needless to say, numerous efforts have been invested to the development of an explicit mathematical solution to the heat conduction equation under the DPL model: Tzou et al. [18] solved the one-dimensional initial-boundary value problem with a surface temperature-jump disturbance through numerically computing the inverse Laplace transformation integral with Riemann sum approximation. Antaki [19] derived a solution for transient temperature in a semi-infinite slab subjected to a constant surface heat flux by first using the Fourier sine transformation to eliminate the space-derivatives and then taking the Laplace transformation to remove the time-derivatives in the heat conduction equation. Tang and Araki [20] obtained the temperature distribution in a finite slab with an energy source near the surface using Green's function method and finite integral transformation technique. Liu [17] relied on a hybrid application of the Laplace transformation method and a control-volume formulation to analyze the DPL effect on the heating in two-layered thin films with an interfacial thermal resistance. Most of these analytical solutions to the DPL heat conduction problems in the literature were formulated ad hoc, only applicable to specific formulations of initial-boundary conditions. Other than the notoriously annoying fictitious numerical oscillations frequently encountered in solving hyperbolic partial differential equations (HPDE), the intrinsic complexity of the DPL heat conduction equation alone (high-order mixed derivative with respect to space and time) poses a tremendous hinder-