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International Journal of Heat and Mass Transfer 51 (2008) 1838–1853

International Journal of HEAT and MASS TRANSFER

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Forced convection heat transfer from an elliptical cylinder to power-law fluids

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Received 21 February 2007; received in revised form 6 June 2007
Available online 4 September 2007

Abstract

Forced convection heat transfer to incompressible power-law fluids from a heated elliptical cylinder in the steady, laminar cross-flow regime has been studied numerically. In particular, the effects of the power-law index ($0.2 \leqslant n \leqslant 1.8$), Reynolds number ($0.01 \leqslant Re \leqslant 40$), Prandtl number ($1 \leqslant Pr \leqslant 100$) and the aspect ratio of the elliptic cylinder ($0.2 \leqslant E \leqslant 5$) on the average Nusselt number (Nu) have been studied. The average Nusselt number for an elliptic cylinder shows a dependence on the Reynolds and Prandtl numbers and power-law index, which is qualitatively similar to that for a circular cylinder. Thus, heat transfer is facilitated by the shear-thinning tendency of the fluid, while it is generally impeded in shear-thickening fluids. The average Nusselt number values have also been interpreted in terms of the usual Colburn heat transfer factor (j). The functional dependence of the average Nusselt number on the dimensionless parameters (Re, n, Pr, E) has been presented by empirically fitting the numerical results for their easy use in process design calculations.

Keywords: Power-law fluids; Elliptical cylinder; Nusselt number; Reynolds number; Prandtl number; Shear-thinning; Shear-thickening

1. Introduction

Owing to their wide ranging applications, considerable research efforts have been devoted to the steady cross-flow of and heat transfer from cylinders of circular and non-circular cross-sections to Newtonian and non-Newtonian fluids. Typical examples include the flow in tubular and pin heat exchangers, hot wire anemometry, sensors and probes, in the RTM process of manufacturing fiber reinforced composites, in filtration screens and aerosol filters, etc. In addition, this flow also represents a classical flow problem in the domain of transport phenomena. Consequently, a voluminous body of information is now available on various aspects of the flow phenomena associated with the transverse flow of Newtonian fluids over a circular cylinder, e.g., see the extensive reviews available in the literature [1–6]. Suffice it to say that adequate information is now available on most aspects of flow and heat transfer for the flow of Newtonian fluids past a circular cylinder. However, it is fair to say that the flow phenomenon has been studied much more extensively than the corresponding heat/mass transfer problems, even for the flow of Newtonian fluids over a circular cylinder.

On the other hand, it is readily acknowledged that many substances of multi-phase nature and/or of high molecular weight encountered in industrial practice (pulp and paper suspensions, food, polymer melts, solutions and in biological process engineering applications, etc.) display shear-thinning and/or shear-thickening behaviour [7]. Owing to their high viscosity levels, these materials are generally processed in laminar flow conditions. Admittedly, many non-Newtonian fluids, notably polymeric systems display viscoelastic behaviour; the available scant literature both for the creeping flow past a single cylinder and over a periodic array of cylinders seems to suggest the viscoelastic effects to be minor in this flow configuration, at least as far as the gross engineering parameters (drag and heat

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doi:10.1016/j.ijheatmasstransfer.2007.06.032

Nomenclature

a	semi-axis of the elliptical cylinder normal to the direction of flow, m
b	semi-axis of the elliptical cylinder along the direction of flow, m
c_p	specific heat of the fluid, J/kg K
E	aspect ratio of the elliptical cylinder, $=b/a$, dimensionless
I_2	second invariant of the rate of the strain tensor, s^{-2}
h	local convective heat transfer coefficient, W/m^2 K
j	Colburn factor for heat transfer, dimensionless
k	thermal conductivity of the fluid, W/m K
m	power-law consistency index, Pa s^n
n	power-law flow behaviour index, dimensionless
$Nu(\theta)$	local Nusselt number, dimensionless
Nu	average Nusselt number, dimensionless
P	pressure, Pa
Pr	Prandtl number, dimensionless
Re	Reynolds number, dimensionless
U_∞	uniform inlet velocity of the fluid, m/s
T	temperature, K
T_∞	temperature of the fluid at the inlet, K
T_w	temperature at the surface of the cylinder, K
U_x, U_y	x- and y-components of the velocity, m/s
x, y	stream-wise and transverse coordinates, m
X_N	normalized average Nusselt number using the corresponding Newtonian value, dimensionless
X_E	normalized average Nusselt number using the corresponding value for circular cylinder, dimensionless

Greek symbols

η	viscosity, Pa s
θ	angular displacement from the front stagnation ($\theta = 0$), degrees
ρ	density of the fluid, kg/m^3
τ	extra stress, Pa

transfer characteristics) are concerned [4]. Therefore, it seems reasonable to begin the analysis with the flow of purely viscous power-law type fluids and the level of complexity can gradually be built up to accommodate the other non-Newtonian characteristics.

As far as known to us, there has been no prior study on the forced convection heat transfer in power-law fluids from an elliptical cylinder. This constitutes the main objective of this work. At the outset, it is desirable, however, to briefly recount the available limited work on the flow of Newtonian and power-law fluids past an elliptical cylinder to facilitate the subsequent presentation of the new results for the forced convection heat transfer in power-law fluids from an elliptical cylinder.

2. Previous work

As noted earlier, while the fluid mechanical aspects of the flow of Newtonian fluids over a circular cylinder have been thoroughly reviewed elsewhere, the corresponding heat transfer literature has been summarized recently by Bharti et al. [8]. In contrast, only limited information is available even for the Newtonian fluid flow over an elliptical cylinder [9].

Even less is known about the power-law fluid flow and heat transfer from a circular cylinder in the steady cross-flow regime. The available literature comprises four creeping flow studies [10–13] and suffice it to say here that all of these analyses are internally consistent with each other. Similarly, only a few studies are available in the steady cross-flow regime relating to finite values of the Reynolds number [9,14–23]. The limits of the cessation of the creeping flow regime and transition from 2D steady symmetric flow to asymmetric flow regimes have been delineated only recently [21] for the flow of power-law fluids across a circular cylinder. This study showed that shear-thickening fluid behaviour can advance the formation of asymmetric wakes. All in all, reliable results are now available for the flow of power-law fluid over a circular cylinder in the two-dimensional steady symmetric flow regime embracing the range of conditions as: $Re \leqslant 40$; $0.2 \leqslant n \leqslant 2$. On the other hand, there have been only three studies [15,17,23] on forced convection heat transfer from a circular cylinder in the steady cross-flow regime. In addition to the local and global heat transfer characteristics, these authors also presented the dependence of the heat transfer on the thermal boundary conditions imposed at the surface of the cylinder. Combined together, these studies encompass the ranges of Reynolds number as $5 \leqslant Re \leqslant 40$, of power-law index $0.6 \leqslant n \leqslant 2$ and Prandtl number as $1 \leqslant Pr \leqslant 100$ and for the two commonly used thermal boundary conditions (i.e., isothermal and isoflux) on the surface of the cylinder. These numerical results have been correlated using simple predictive expressions to permit an easy estimation of the value of the average heat transfer coefficient for a cylinder immersed in streaming power-law liquids. The available scant experimental studies have also been summarized elsewhere [2–4,24] and these are not repeated here. Suffice it to add here that the preliminary comparisons between the predictions and the scant mass transfer results are encouraging.

In contrast, as far as known to us, there has only one study dealing with the steady flow of power-law fluids over an elliptical cylinder [9]. Extensive results were presented on the individual and total drag coefficients, streamline and surface pressure profiles and their functional dependence