International Journal of Heat and Mass Transfer 52 (2009) 5792–5795

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

The Cheng–Minkowycz problem for natural convective boundary-layer flow in a porous medium saturated by a nanofluid

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ARTICLE INFO

Article history:
Received 14 April 2009
Received in revised form 31 July 2009
Accepted 31 July 2009
Available online 11 September 2009

Keywords:
Cheng–Minkowycz problem
Natural convection
Nanofluid
Porous medium
Similarity solution

ABSTRACT

The Cheng–Minkowycz problem of natural convection past a vertical plate, in a porous medium saturated by a nanofluid, is studied analytically. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. For the porous medium the Darcy model is employed. A similarity solution is presented. This solution depends on a Lewis number Le, a buoyancy-ratio number Nr, a Brownian motion number Nb, and a thermophoresis number Nt. The dependency of the Nusselt number on these four parameters is investigated.

1. Introduction

The term “nanofluid“ refers to a liquid containing a dispersion of submicronic solid particles (nanoparticles). The term was coined by Choi [1]. The characteristic feature of nanofluids is thermal conductivity enhancement, a phenomenon observed by Masuda et al. [2]. This phenomenon suggests the possibility of using nanofluids in advanced nuclear systems (Buongiorno and Hu [3]).

A comprehensive survey of convective transport in nanofluids was made by Buongiorno [4], who says that a satisfactory explanation for the abnormal increase of the thermal conductivity and viscosity is yet to be found. He focused on the further heat transfer enhancement observed in convective situations. Buongiorno notes that several authors have suggested that convective heat transfer enhancement could be due to the dispersion of the suspended nanoparticles but he argues that this effect is too small to explain the observed enhancement. Buongiorno also concludes that turbulence is not affected by the presence of the nanoparticles so this cannot explain the observed enhancement. Particle rotation has also been proposed as a cause of heat transfer enhancement, but Buongiorno calculates that this effect is too small to explain the effect. With dispersion, turbulence and particle rotation ruled out as significant agencies for heat transfer enhancement, Buongiorno proposed a new model based on the mechanics of the nanoparticle/base-fluid relative velocity.

Buongiorno [4] noted that the nanoparticle absolute velocity can be viewed as the sum of the base fluid velocity and a relative velocity (that he calls the slip velocity). He considered in turn seven slip mechanisms: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity settling. After examining each of these in turn, he concluded that in the absence of turbulent effects it is the Brownian diffusion and the thermophoresis that will be important. Buongiorno proceeded to write down conservation equations based on these two effects.

The problem of natural convection in a porous medium past a vertical plate is a classical problem first studied by Cheng and Minkowycz [5]. The problem is presented as a paradigmatic configuration and solution in the book by Bejan [6]. The extension to the case of heat and mass transfer was made by Bejan and Khair [7]. Further work on this topic is surveyed in Sections 5.1 and 9.2.1 in Nield and Bejan [8]. A review of the heat transfer characteristics of nanofluids has been made by Wang and Mujumdar [9].

In the present paper the model of [4] is applied to the problem in [5].

2. Analysis

It is assumed that nanoparticles are suspended in the nanofluid using either surfactant or surface charge technology. This prevents particles from agglomeration and deposition on the porous matrix.

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doi:10.1016/j.ijheatmasstransfer.2009.07.024

Nomenclature

D_B	Brownian diffusion coefficient
D_T	thermophoretic diffusion coefficient
f	rescaled nanoparticle volume fraction, defined by Eq. (20)
$\mathbf{g}$	gravitational acceleration vector
k_m	effective thermal conductivity of the porous medium
K	permeability of the porous medium
Le	Lewis number, defined by Eq. (27)
Nr	buoyancy ratio, defined by Eq. (24)
Nb	Brownian motion parameter, defined by Eq. (25)
Nt	thermophoresis parameter, defined by Eq. (26)
Nu	Nusselt number, defined by Eq. (32)
Nur	reduced Nusselt number, $\mathrm{Nu}/\mathrm{Ra}_x{}^{1/2}$
p	pressure
q''	wall heat flux
Ra_x	local Rayleigh number, defined by Eq. (18)
s	dimensionless stream function, defined by Eq. (20)
T	temperature
T_w	temperature at the vertical plate
T_∞	ambient temperature attained as y tends to infinity
$\mathbf{v}$	Darcy velocity, (u, v)
(x, y)	Cartesian coordinates (x-axis is aligned vertically upwards, plate is at $y = 0$)

Greek symbols

α_m	thermal diffusivity of the porous medium, $\frac{k_m}{(\rho c)_f}$
β	volumetric expansion coefficient of the fluid
ε	porosity
η	similarity variable, defined by Eq. (19)
θ	dimensionless temperature, defined by Eq. (20)
μ	viscosity of the fluid
ρ_f	fluid density
ρ_p	nanoparticle mass density
$(\rho c)_f$	heat capacity of the fluid
$(\rho c)_m$	effective heat capacity of the porous medium
$(\rho c)_p$	effective heat capacity of the nanoparticle material
τ	parameter defined by Eq. (13), $\frac{\varepsilon(\rho c)_p}{(\rho c)_f}$
ϕ	nanoparticle volume fraction
ϕ_w	nanoparticle volume fraction at the vertical plate
ϕ_∞	ambient nanoparticle volume fraction attained as y tends to infinity
ψ	stream function, defined by Eq. (14)

We consider a two-dimensional problem. We select a coordinate frame in which the x-axis is aligned vertically upwards. We consider a vertical plate at $y = 0$. At this boundary the temperature T and the nanoparticle fraction ϕ take constant values T_w and ϕ_w, respectively. The ambient values, attained as y tends to infinity, of T and ϕ are denoted by T_∞ and ϕ_∞, respectively.

The Oberbeck-Boussinesq approximation is employed. Homogeneity and local thermal equilibrium in the porous medium is assumed. We consider a porous medium whose porosity is denoted by ε and permeability by K. The Darcy velocity is denoted by $\mathbf{v}$. The following four field equations embody the conservation of total mass, momentum, thermal energy, and nanoparticles, respectively. The field variables are the Darcy velocity $\mathbf{v}$, the temperature T and the nanoparticle volume fraction ϕ.

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\frac{\rho_f}{\varepsilon}\frac{\partial \mathbf{v}}{\partial t} = -\nabla p - \frac{\mu}{K}\mathbf{v} + [\phi\rho_p + (1-\phi)\{\rho_f(1-\beta(T-T_\infty))\}]\mathbf{g}, \tag{2}$$

$$(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f \mathbf{v}\cdot\nabla T = k_m \nabla^2 T + \varepsilon(\rho c)_p [D_B \nabla\phi \cdot \nabla T + (D_T/T_\infty)\nabla T \cdot \nabla T], \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{\varepsilon}\mathbf{v}\cdot\nabla\phi = D_B \nabla^2\phi + (D_T/T_\infty)\nabla^2 T. \tag{4}$$

We write $\mathbf{v} = (u, v)$.

Here ρ_f, μ and β are the density, viscosity, and volumetric volume expansion coefficient of the fluid while ρ_P is the density of the particles. The gravitational acceleration is denoted by $\mathbf{g}$. We have introduced the effective heat capacity $(\rho c)_m$, and the effective thermal conductivity k_m of the porous medium. The coefficients that appear in Eqs. (3) and (4) are the Brownian diffusion coefficient D_B and the thermophoretic diffusion coefficient D_T. Details of the derivation of Eqs. (3) and (4) are given in the papers by Buongiorno [4], Tzou [10,11] and Nield and Kuznetsov [12,13]. The flow is assumed to be slow so that an advective term and a Forchheimer quadratic drag term do not appear in the momentum equation.

The boundary conditions are taken to be

$$v = 0, \quad T = T_w, \phi = \phi_w \text{ at } y = 0, \tag{5}$$

$$u = v = 0, \quad T \to T_\infty, \phi \to \phi_\infty \text{ as } y \to \infty. \tag{6}$$

We consider a steady state flow.

In keeping with the Oberbeck-Boussinesq approximation and an assumption that the nanoparticle concentration is dilute, and with a suitable choice for the reference pressure, we can linearize the momentum equation and write Eq. (2) as

$$0 = -\nabla p - \frac{\mu}{K}\mathbf{v} + [(\rho_p - \rho_{f\infty})(\phi - \phi_\infty) + (1-\phi_\infty)\rho_{f\infty}\beta(T - T_\infty)]\mathbf{g}. \tag{7}$$

We now make the standard boundary-layer approximation, based on a scale analysis, and write the governing equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{8}$$

$$\frac{\partial p}{\partial x} = -\frac{\mu}{K}u + \left[(1-\phi_\infty)\rho_{f\infty}\beta g(T-T_\infty) - (\rho_p - \rho_{f\infty})g(\phi - \phi_\infty)\right] \tag{9}$$

$$\frac{\partial p}{\partial y} = 0, \tag{10}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \nabla^2 T + \tau\left[D_B \frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{11}$$

$$\frac{1}{\varepsilon}\left(u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y}\right) = D_B \frac{\partial^2 \phi}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}. \tag{12}$$

where

$$\alpha_m = \frac{k_m}{(\rho c)_f}, \quad \tau = \frac{\varepsilon(\rho c)_p}{(\rho c)_f}. \tag{13}$$

One can eliminate p from Eqs. (9) and (10) by cross-differentiation. At the same time one can introduce a stream function ψ defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \tag{14}$$

so that Eq. (8) is satisfied identically.We are then left with the following three equations.

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{(1-\phi_\infty)\rho_{f\infty}\beta g K}{\mu}\frac{\partial T}{\partial y} - \frac{(\rho_p - \rho_{f\infty})gK}{\mu}\frac{\partial \phi}{\partial y} \tag{15}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \alpha_m \nabla^2 T + \tau\left[D_B \frac{\partial \phi}{\partial y}\frac{\partial T}{\partial y} + \left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial T}{\partial y}\right)^2\right], \tag{16}$$

$$\frac{1}{\varepsilon}\left(\frac{\partial \psi}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \phi}{\partial y}\right) = D_B \frac{\partial^2 \phi}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2}. \tag{17}$$