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Simulating oscillatory flows in Rayleigh–Bénard convection using the lattice Boltzmann method

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Abstract

Rayleigh–Bénard convection is a fundamental phenomenon found in many atmospheric and industrial applications. Many numerical methods have been applied to analyze this problem, including the lattice Boltzmann method (LBM), which has emerged as one of the most powerful computational fluid dynamics (CFD) methods in recent years. Using a simple LB model with the Boussinesq approximation, this study investigates the 2D Rayleigh–Bénard problem from the threshold of the primary instability with a theoretical value of critical Rayleigh number $Ra_c = 1707.76$ to the regime near the flow bifurcation to the secondary instability. Since the fluid of LBM is compressible, an appropriate velocity scale for natural convection, i.e. $V \equiv \sqrt{\beta g_y \Delta T H}$, is carefully chosen at each value of the Prandtl number to ensure that the simulations satisfy the incompressible condition. The simulation results show that periodic unsteady flows take place at certain Prandtl numbers with an appropriate Rayleigh number. Furthermore, the Nusselt number is found to be relatively insensitive to the Prandtl number in the current simulation ranges of $0.71 \leqslant Pr \leqslant 70$ and $Ra \leqslant 10^5$. Finally, the relationship between the Nusselt number and the Rayleigh number is also investigated.

Keywords: Rayleigh–Bénard convection; Natural convection; Lattice Boltzmann method (LBM); Computational fluid dynamics (CFD)

1. Introduction

Due to its practical importance in many general science and engineering applications, Rayleigh–Bénard convection has been the subject of many theoretical, experimental, and numerical studies. Since Rayleigh–Bénard convection presents the evolution from the stationary state to the fully developed turbulent regime with many different flow patterns and sequences of bifurcations, it is widely investigated as the problems of different transition mechanisms in hydrodynamics [1–4].

In Rayleigh–Bénard convection, the primary instability, which represents a transition from diffusive thermal conduction to stationary time-independent steady convection with a structure of steady 2D rolls, occurs at a critical Rayleigh number of $Ra_c = 1707.76$ for the case of no-slip boundary conditions imposed on solid walls. The value of this critical Rayleigh number is independent of the Prandtl number. However, as the Rayleigh number increases, a bifurcation to a time-dependent flow structure with a single-frequency periodic state is observed, namely the secondary instability. This transition to the secondary instability is strongly dependent on the Prandtl number. Moreover, as the Rayleigh number is increased further, two-frequency quasi-periodic flow is generated from the single-frequency oscillatory state and the flow finally transits to a chaotic state in the fully developed turbulent regime. Early experimental results for the transition to turbulence in Rayleigh–Bénard convection were presented by Krishnamurti [5]. More recently, various studies have employed numerical methods to investigate the bifurcations to oscillatory flow in Rayleigh–Bénard convection [6–10]. In these studies, the authors presented numerical results for symmetry-breaking solutions of flow

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Nomenclature

a_c	critical wave number for primary instability in Rayleigh–Bénard convection
AR	aspect ratio
c_i	microscopic particle velocity in each lattice link i
c_s	speed of sound
f	distribution function for the flow field
f_D	reference thermal diffusive frequency
f_P	induced oscillatory frequency
f^*	dimensionless frequency ratio
g	distribution function for the temperature field
g_y	acceleration of gravity in the y-direction
H	vertical height of the computational domain
$\dot{J}_i$	momentum input from the buoyant body force in each lattice link i
L	horizontal length of the computational domain
Nu	Nusselt number
$\overline{Nu}$	average Nusselt number
Pr	Prandtl number
r	power value for the power law: $Nu \propto Ra^r$
Ra	Rayleigh number
Ra_c	critical Rayleigh number for primary instability in Rayleigh–Bénard convection
T	temperature
t	macroscopic time
t_D	reference thermal diffusive time scale: $t_D = H^2/\alpha$
t_p	time of period for the induced oscillatory flow
u_A	macroscopic flow velocities, where subindex A is the components of Cartesian coordinates
V	characteristic velocity of natural convection, $V \equiv \sqrt{\beta g_y \Delta T H}$

Greek symbols

α	thermal diffusivity
β	expansion coefficient: $\beta \equiv -1/\rho_{\mathrm{ref}}(\partial\rho/\partial T)_P$
ℓ	length scale according to specific case of natural convection problems
ΔT	temperature difference
Δt	time interval (step) of LBM
ν	kinetic viscosity
ρ	fluid density
τ_D	relaxation time for the temperature field
τ_ν	relaxation time for the flow field

bifurcations and estimated the Rayleigh number for the oscillatory flows occurring at Prandtl numbers of approximately $Pr = 6$.

The kinetic-based lattice Boltzmann method (LBM) is a powerful numerical technique for simulating fluid flows and modeling the physics in fluids [11–15]. However, the application of LBM to heat transfer problems has not achieved great success for the thermal models due to the severe numerical instability caused by breaking the isothermal condition [15]. Various numerical simulations have been performed using different thermal LB models or Boltzmann-based schemes to investigate 2D Rayleigh–Bénard convection [16–19]. Although the results provided by these studies for stationary convection are in good agreement with the data presented in [2], the flow bifurcation to the secondary instability at different Prandtl numbers was not examined and discussed by thermal LB models.

The present study employs a simple thermal LB model with the Boussinesq approximation to simulate the oscillatory flows of the secondary instability in 2D Rayleigh–Bénard convection. The present study also investigates the structure of the oscillatory flow for this natural convection problem using the simple LB model. In this study, the thermal LB model is simplified by neglecting the viscous thermal dissipation for incompressible flows. However, the compressibility of LB fluids must be taken into consideration in the applications of compressible codes to the incompressible limit, as reported in [16,17,20]. Therefore, a correction procedure is applied to obtain an appropriate characteristic velocity of natural convection, i.e. $V \equiv \sqrt{\beta g_y \Delta T H}$, for different Prandtl numbers to ensure that the incompressible condition is satisfied. Furthermore, the simulations are restricted to the Prandtl number range of $0.71 \leqslant Pr \leqslant 70$ and to the Rayleigh numbers of $Ra \leqslant 10^5$ to ensure numerical stability and computational accuracy. Having explored the oscillatory flows of the secondary instability in 2D Rayleigh–Bénard convection, the relationship between the Nusselt number and the Rayleigh number is investigated for Prandtl numbers in the range of $0.71 \leqslant Pr \leqslant 70$.

2. Numerical method

2.1. Lattice Boltzmann model

In investigating the natural convection problem, this study neglects the viscous heat dissipation in applications of incompressible flow such that a simple lattice Boltzmann method can be used. The LB model comprises two distribution functions, f and g, for the flow filed and the temperature field, respectively. The density and the temperature distribution functions, f and g, are defined as the probability of particles at site x at time t moving with the particle velocity c_i during the time interval Δt in each lattice direction (link) i. The same model was proposed in [21,22]. The two distribution functions obey their respective lattice Boltzmann transport equations with the single relaxation Bhatnagar–Gross–Krook (BGK) approximation, i.e.