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Thermally developing Brinkman–Brinkman forced convection in rectangular ducts with isothermal walls

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Abstract

The Extended Weighted Residuals Method (EWRM) is applied to investigate the effects of viscous dissipation on the thermal development of forced convection in a porous-saturated duct of rectangular cross-section with isothermal boundary condition. The Brinkman flow model is employed for determination of the velocity field. The temperature in the flow field was computed by utilizing the Green's function solution based on the EWRM. Following the computation of the temperature field, expressions are presented for the local Nusselt number and the bulk temperature as a function of the dimensionless longitudinal coordinate. In addition to the aspect ratio, the other parameters included in this computation are the Darcy number, viscosity ratio, and the Brinkman number.
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1. Introduction

Flow through porous media is important in numerous engineering applications including geothermal energy, petroleum reservoirs, nuclear reactors, drying, and fuel cells. Almost all of the natural porous media are associated with such small porosity that the Darcy flow model is applicable. However, for man-made porous media with higher porosity, the Brinkman model predicts hydraulics through such hyperporous media, as noted by Nield and Bejan [1].

Because of the use of the so-called hyperporous media in the cooling of electronic equipment, there has recently been renewed interest in the problem of forced convection in a porous medium channel. However, the literature on the effects of viscous dissipation on thermal development is limited to work pertaining to parallel plate channel [2–5] or circular tube [6–9]. In some of these articles the velocity distribution is slug type while in others the boundary and shear effects are included via a Brinkman term to form a Brinkman–Brinkman problem. The term 'Brinkman–Brinkman', proposed by Nield [10], refers to a problem involving a saturated porous medium in which the momentum transfer is modeled by a Brinkman equation [11], and the thermal energy equation includes a viscous dissipation term involving a Brinkman number [12]. The problem becomes more complicated when one seeks analytical solutions for a thermally developing Brinkman–Brinkman problem through ducts of arbitrary cross-section. For two-dimensional ducts, the complexity of the problems become clearer when one observes that even fully developed solutions, with or without the effects of viscous dissipation, are limited to the work reported in [13–16]. The studies of the thermally developing forced convection heat transfer in elliptical ducts in [17] and for ducts with rectangular cross-sections [18] are without inclusion of the

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Nomenclature

A	area (m^2)
$\mathbf{A}$	matrix
a	duct dimension, see Fig. 1
a_{ij}	elements of matrix $\mathbf{A}$
$\mathbf{B}$	matrix
B_m	coefficients
b	duct dimension, see Fig. 1
b_{ij}	elements of matrix $\mathbf{B}$
Br	Brinkman number, $\mu_e U^2/[k_e(T_1-T_2)]$
c_p	constant pressure specific heat (J/kg K)
$\mathbf{D}$	matrix
Da	Darcy number (K/a^2)
D_h	hydraulic diameter $4ab/(a+b)$ (m)
d_{mj}	elements of matrix $\mathbf{D}$
$\mathbf{E}$	matrix with elements e_{ij}
e_{ij}	elements of matrix $\mathbf{E}$
f_i, f_j	basis functions
G	Green's function
h	heat transfer coefficient (W/m^2 K)
$\bar{h}$	average heat transfer coefficient (W/m^2 K)
i, j	indices
K	permeability (m^2)
k_e	effective thermal conductivity (W/m K)
M	viscosity ratio, μ_e/μ
m, n	indices
Nu_D	local Nusselt number, hD_h/k_e
$\bar{N}u_D$	average Nusselt number, $\bar{h}D_h/k_e$
$\mathbf{P}$	matrix having elements p_{mi}
Pe	Péclet number, $\rho c_p Ua/k_e$
Pr	Prandtl number, $\mu_e c_p/k_e$
p	pressure (Pa)

p_{mi}	elements of matrix $\mathbf{P}$
Re_D	Reynolds number, $\rho UD_h/\mu_e$
S	volumetric heat source, Eq. (4b) (W/m^3)
S^*	dimensionless heat source, Eq. (18b)
T	temperature (K)
T_1	temperature at $x=0$ (K)
T_2	wall temperature (K)
U	average velocity (m/s)
u	velocity (m/s)
$\bar{u}$	dimensionless velocity, $-\mu u/(a^2\partial p/\partial x)$
$\hat{u}$	u/U
x	axial coordinate (m)
$\bar{x}$	$(x/a)/Pe$
y, z	coordinates (m)
$\bar{y}, \bar{z}$	y/a and z/a

Greek symbols

θ	dimensionless temperature
λ_m	eigenvalues
μ	fluid viscosity (N s/m^2)
μ_e	effective viscosity (N s/m^2)
ξ	dummy variable of integration
ρ	fluid density (kg/m^3)
ψ	eigenfunction

Subscripts

b	bulk
e	effective
o	unheated length
w	wall

viscous dissipation effects. In a recent work, Haji-Sheikh et al. [19] have considered the effects of viscous dissipation on heat transfer in the entrance region of ducts of arbitrary cross-section with a special attention to the isosceles triangular case.

Earlier work on the effects of viscous dissipation in ducts, clear of solid material, is surveyed by Shah and London [20] and for in porous media surveyed by Magyari et al. [22]. This paper treats the more general case of thermally developing forced convection in rectangular ducts wherein the viscous dissipation is significant. The EWRM in an extended form, as discussed in [19], is the selected computational methodology. This study treats the case of a duct of rectangular cross-section with walls held at a constant and uniform temperature, i.e. the **T** boundary condition in the terminology of Shah and London [20], which is appropriate when the thermal conductivity of the enclosing walls is sufficiently high. Here, the Green's function solution in [18] is modified mainly to account for the viscous dissipation effects on the thermal development. For the case of the Darcy flow model, the hydrodynamically developed velocity profile is that of slug flow, and the problem is mathematically similar to a pure conduction [21], but this paper considers the more complicated flow appropriate to the Brinkman model.

2. Analysis

2.1. Fluid flow analysis

For a passage with a constant but arbitrarily shaped cross-section, based on the ligament dimension, the Brinkman momentum equation, is

$$\mu_e\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\mu}{K}u-\frac{\partial p}{\partial x}=0. \tag{1}$$

By selecting $\bar{y}=y/a$, $\bar{z}=z/a$, and $\bar{u}=-\mu u/(a^2\partial p/\partial x)$, the dimensionless form of Eq. (1) becomes,

$$M\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right)-\frac{1}{Da}\bar{u}+1=0, \tag{2}$$

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