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International Journal of Heat and Mass Transfer 49 (2006) 3681–3690

International Journal of HEAT and MASS TRANSFER

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On temperature prediction at low Re turbulent flows using the Churchill turbulent heat flux correlation

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Received 4 November 2005; received in revised form 9 February 2006
Available online 17 April 2006

Abstract

The present work investigates the prediction of mean temperature profiles in turbulent channel flow using the fraction of the heat flux due to turbulence. According to this new model, suggested by Churchill and co-workers, fully developed flow and convection can be expressed as local fractions of the shear stress and the heat flux density due to turbulent fluctuations, respectively. The fully developed temperature profile can be predicted if the velocity field and the turbulent Prandtl number are known. Temperature profiles for *Pr* between 0.01 and 50,000 have been obtained theoretically and with simulations through the use of Lagrangian methods for both plane Poiseuille flow and plane Couette flow. The half channel height for all simulations was $h = 150$ in wall units. The theoretical predictions have been found to agree with the data quite well for a range of *Pr*, but there are deviations at very high *Pr*.

Keywords: Prandtl number; Turbulent transport; Lagrangian methods; Algebraic models

1. Introduction

Scaling questions about the law of the wall (a bulwark of turbulence theory) in low and intermediate Reynolds numbers (i.e., flow in channels and pipes, instead of infinite boundary layers) have been raised for the velocity field [1–7]. It is now argued that turbulence quantities do not scale with the viscous wall parameters, as defined conventionally, and that a Reynolds number effect is present. Similar issues can be raised for the equivalent of the law of the wall for heat transfer. Results from direct numerical simulations have shown that scaling with the wall parameters for low Reynolds numbers does not provide universal behavior for the fluctuating thermal field [8–10]. However, an effort to explore the scaling of heat transfer similar to that for momentum has not been vigorously pursued, with the notable exception of Churchill and coworkers [11–14] who suggested the use of the local fraction of the heat flux density due to turbulent fluctuations to predict the mean temperature, introducing, thus, the use of a scale different than the conventional friction temperature.

An algebraic model for the prediction of mean turbulence quantities was first introduced by Churchill and Chan [11] in 1995; it has been suggested to be superior in several aspects when compared with other conventional algebraic models that are based on empiricisms and approximations. Heuristic concepts, like the eddy diffusivity or the mixing length that are not fundamentally sound, are totally avoided. According to this new model, fully developed flow and convection can be expressed as fractions, respectively, of shear stress and heat flux density due to turbulent fluctuations. The mean temperature profile can be predicted based on exact equations, given the velocity profile and the turbulent Prandtl number. This is a very significant

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0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2006.02.022

Nomenclature

C_f	correction factor defined in Eqs. ((22))
C_p	specific heat at constant pressure (kJ/(kg K))
E_α	eddy diffusivity (kg/(m s))
E_v	eddy viscosity (kg/(m s))
h	half height of the channel (m)
k	thermal conductivity (W/(m K))
P_1	conditional probability for a marker to be at a location (x, y) at time t, given that it was released at a known time from a known location at the channel wall
P_2	joint probability for a marker to be at a location (x, y)
Pr	Prandtl number, $Pr = \nu/\alpha$
Pr_t	turbulent Prandtl number, $Pr_t = E_v/E_\alpha$
q	heat flux (kW/m^2)
Re	Reynolds number, $Re = U_c h/\nu$
T	temperature (K)
T^*	friction temperature, $T^* = q_w/(\rho C_p u^*)$
$\overline{T}$	mean temperature (K)
T'	temperature fluctuation (K)
$\overline{T'v'}$	normal heat flux
t	time (s)
t_0, t_f	initial and final time of tracking markers (s)
U	velocity (m/s)
$\overline{U}$	mean velocity (m/s)
$u*$	friction velocity, $u^* = (\tau_w/\rho)^{1/2}$ (m/s)
$\overline{u'v'}$	normal shear stress
x, y, z	streamwise, normal and spanwise coordinates
X, Y	Lagrangian displacement of a marker from the source in the x, y directions

Greek symbols

α	thermal diffusivity (m^2/s)
γ	correction term defined in Eq. (6)
ν	kinematic viscosity (m^2/s)
π	trigonometric pi ($\pi = 3.14159\ldots$)
ρ	fluid density (kg/m^3)
σ	standard deviation of the pdf that describes the diffusive motion of the heat markers
τ	shear stress (Pa)

Superscripts and subscripts

$\overline{()}$	ensemble average
$\vec{()}$	vector quantity
$()^+$	value made dimensionless with the wall parameters
$()^{++}$	local fraction due to turbulence
$()^*$	friction value
$()_0$	value at the instant of marker release
$()_w$	value at the wall of the channel

contribution in the area of turbulent convection, especially given the semi-empirical predictive capabilities of the past [15]. In addition, the concept of a scale that is directly associated with turbulence, like the fraction of the local heat flux due to turbulence suggested by Churchill, seems more natural, when contrasted to scaling based on the wall friction temperature (which is dependent only on viscous effects). In other words, since the temperature fluctuations are generated due to velocity fluctuations and their production occurs within the conductive wall layer but not at the wall, it makes sense to predict turbulent transfer based on a turbulence quantity rather than a viscous one.

Churchill et al. [16] have recently conducted an analysis of the sensitivity of the new algebraic model to the numerical empiricisms and empirical functions that enter into it. They found that predictions are rather insensitive to reasonable changes in the empirical parameters of the model. On the other hand, comparison of the model predictions to either simulation results or experimental measurements for a truly extensive range of data has not been reported. This is the space that the present paper covers: the verification of the Churchill model for a range of fluids (i.e., a range of Prandtl numbers) and for fundamentally different turbulent velocity fields (i.e., pressure driven and shear driven). Our research group has used direct numerical simulation (DNS) in conjunction with Lagrangian scalar tracking (LST) to study turbulent transport for an extensive range of Prandtl numbers (from $Pr = 0.01$ to 50,000) in Poiseuille channel flow [17,18] and in Couette flow [18,19]. Mean temperature profile predictions through this method agree very well with previous experimental and direct numerical simulation results, and, quite importantly, they have been obtained with a consistent methodology that has been used for a range of cases where conventional Eulerian direct simulations are not yet feasible.

2. Background and theory

2.1. Eulerian heat transfer

In the Eulerian framework, the temperature T can be decomposed into the mean temperature $\overline{T}$ and the fluctuation T'. The temperature is conventionally made dimensionless by normalizing with the friction temperature T^*, $T^* = q_w/(\rho C_p u^*)$, where q_w is the heat flux at the wall defined in terms of the thermal conductivity of the fluid k as

$$q_w = -k\left(\frac{d\overline{T}}{dy}\right)_w \tag{1}$$

Therefore, a dimensionless temperature T^+ can be calculated by

$$T^+ = \frac{T}{T^*} = -\frac{T\rho C_p u^*}{k\left(\frac{d\overline{T}}{dy}\right)_w} = -Pr\frac{T}{\left(\frac{d\overline{T}}{dy^+}\right)_w} \tag{2}$$