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Combined forced and free flow in a vertical circular duct subjected to non-axisymmetric wall heating conditions

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Abstract

The fully developed mixed convection flow in a vertical circular duct is investigated analytically, under the assumption of laminar parallel flow. A wall heat flux uniform in the axial direction and dependent on the angular coordinate is considered. As a consequence, the fluid temperature is three dimensional, since it changes in the radial, axial and angular directions. An analytical method based on Fourier series expansions of temperature and velocity fields is adopted to determine the velocity and the temperature distributions as well as the friction factor and the average Nusselt number. The general solution, expressed in terms of Bessel functions, is applied to study a case that has a special importance in technical applications: a duct whose wall is half subject to a uniform heat flux and half adiabatic. The positive and negative threshold values of the ratio between the Grashof number *Gr* and the Reynolds number *Re* for the onset of the flow reversal phenomenon are determined. A comparison between the average Nusselt number for the considered non-axisymmetric case and that for the case of a duct subject to a uniform wall heat flux is performed.

Keywords: Non-axisymmetric boundary conditions; Circular duct; Mixed convection; Laminar flow; Analytical methods

1. Introduction

Laminar mixed convection in vertical or inclined circular ducts is a subject that has been extensively studied over the last decades. For instance, the review papers [1,2], as well as the references therein, show the most important results achieved on this subject. In fact, most papers on this topic refer to axisymmetric thermal boundary conditions. For example, in [3] the Author discussed the fully developed mixed convection in a vertical tube in the case of laminar flow with a uniform wall heat flux. However, there are several technical cases such that the wall temperature and the wall heat flux depend on the angular coordinate. Non-axisymmetric thermal boundary conditions have been studied, for instance, in [4] and, more recently, in [5]. In [6], the Authors studied the case of a horizontal duct with half cross-section subject to a uniform wall heat flux and the other half adiabatic.

In the present paper, mixed convection flow in a vertical circular duct is studied with reference to a wall heat flux which is uniform along the axial direction and is an arbitrary function of the angular coordinate. Therefore, a net fluid heating occurs in the flow direction. The fully developed region is studied and laminar parallel flow is considered. Moreover, the Boussinesq approximation is applied by assuming the axially varying average temperature in a duct section as the reference fluid temperature. As it has been shown in [7], this assumption is the best choice to ensure the validity of the Boussinesq approximation. The momentum and energy balance equations are written in a dimensionless form and are solved by employing an analytical method based on Fourier series expansions of both the temperature field and the velocity field with respect to the angular coordinate ϑ. The velocity field, the temperature field, the friction factor and the average Nusselt number are evaluated. A special case is studied in detail: the case

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Nomenclature

A	dimensionless parameter, defined by Eq. (24)
$b_0(r)$, $a_n(r)$, $b_n(r)$	Fourier series coefficients, employed in Eq. (20)
$h_0(r)$, $c_n(r)$, $h_n(r)$	Fourier series coefficients, employed in Eq. (21)
f	Fanning friction factor, defined by Eq. (54)
$F(\vartheta)$	dimensionless arbitrary function of ϑ, defined by Eq. (19)
$\mathbf{g}$	gravitational acceleration vector
g	magnitude of the gravitational acceleration
Gr	Grashof number, defined in Eq. (6)
I_n, J_n	Bessel functions of order n
k	thermal conductivity of the fluid
m,n	positive integers
$\overline{Nu}$	average Nusselt number, defined by Eq. (62)
$\overline{Nu}_{sym}$	average Nusselt number for uniform wall heat flux
P	difference between the pressure and the hydrostatic pressure
$q_w(\vartheta)$	local wall heat flux per unit area
$\overline{q}_w$	average value of q_w
r	dimensionless radial coordinate, defined in Eq. (6)
R	radial coordinate
R_0	radius of the duct
Re	Reynolds number, defined in Eq. (6)
t	dimensionless temperature, defined in Eq. (6)
t_b	dimensionless bulk temperature, defined by Eq. (60)
t_{peak}	peak value of the dimensionless temperature
$\overline{t}_w$	dimensionless average wall temperature, defined by Eq. (64)
T	temperature
T_b	bulk temperature of the fluid, defined by Eq. (59)
T_0	average fluid temperature in a duct section, defined by Eq. (7)
$\overline{T}_w$	average wall temperature
u	dimensionless velocity, defined in Eq. (6)
$\mathbf{U}$	fluid velocity vector
U	X-component of the fluid velocity
U_0	average velocity in a duct section, defined by Eq. (9)
W_n	function of A, defined by Eq. (45)
X	axial coordinate

Greek symbols

α	thermal diffusivity
β	volumetric coefficient of thermal expansion
γ_n	Fourier series coefficients, employed in Eq. (19)
ΔT	reference temperature difference
ϑ	angular coordinate
λ	dimensionless pressure drop, defined in Eq. (6)
Λ_1	positive threshold value of Gr/Re
Λ_2	negative threshold value of Gr/Re
μ	dynamic viscosity
ν	kinematic viscosity
ρ	mass density
ρ_0	mass density for $T = T_0$
$\overline{\tau}_w$	average wall shear stress, defined by Eq. (55)
ω_n	Fourier series coefficients, employed in Eq. (19)

of a vertical circular duct having a wall which is half subject to a uniform heat flux and half adiabatic. The solution shows that the velocity profile can be strongly influenced by the buoyancy forces and may display flow reversal phenomena. Plots of both the dimensionless temperature and the dimensionless velocity as functions of the angular coordinate ϑ and of the dimensionless radial coordinate r are presented for some values of the ratio between the Grashof number Gr and the Reynolds number Re. Moreover, an analysis of the conditions for the occurrence of the flow reversal phenomenon is performed and the positive and negative threshold values of Gr/Re are determined. Finally, the average Nusselt number is compared with that of a duct subject to a uniform axisymmetric wall heat flux.

2. Mathematical model

Let us consider a vertical circular duct with radius R_0 and a cylindrical coordinate system (X, R, ϑ), as sketched in Fig. 1. Let us suppose that a Newtonian fluid flows

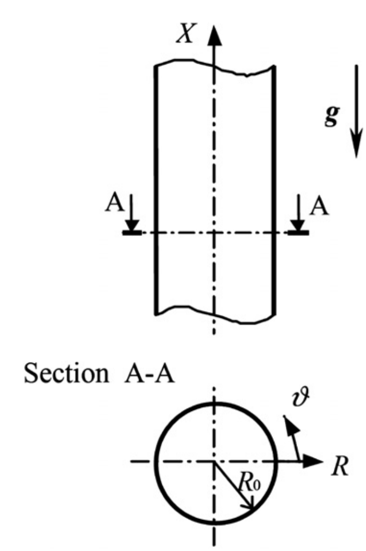

Fig. 1. The vertical circular duct and the cylindrical coordinate system.