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# Analytical modeling of electrokinetic effects on flow and heat transfer in microchannels

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**Abstract**

A fundamental understanding of electrolytic flow in microchannels is essential for the design of microfluidic devices. Hence, an analytic investigation is presented on the effects of electrostatic potential in microchannels. Solving the Navier–Stokes equations, an expression for the $C_f Re$ product is presented. Solving the energy equation the Nusselt number for constant wall heat flux and constant wall temperature boundary conditions are presented with analytic expressions over a wide range of operating conditions.

*Keywords:* Electric double layer; Electro-viscous effect; Friction factor; Nusselt number

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## 1. Introduction

Micro-electro-mechanical-systems (MEMS) and other microfluidic technologies have revolutionized many aspects of applied sciences and engineering, such as heat exchangers [1], pumps [2], combustors, gas absorbers, solvent extractors, fuel processors [3], and on-chip biomedical and biochemical analysis instruments [4]. All of these devices involve fluid flow and heat transfer in microchannels. Indeed, much study has been focused on microchannels [5] for efficient cooling of chips due to their very high heat transfer coefficients [6].

When dealing with micro-fluidics or flows in microchannels, the interfacial effects (phenomenon happening at the surface of the microchannel), which are negligible in bulk fluid flows, becomes more pronounced [7–13]. In particular, electrolytic flow in microchannels can be significantly different than non-electrolytic flows. The phenomenon manifests itself by generating a viscous effect, which affects both the flow and heat transfer.

The present work deals with the modification of Navier–Stokes equation to take into account the effect of electrokinetics and the development of analytic expressions for both the friction factor and Nusselt number. Previous investigations have ignored these effects at lower ionic concentrations. The present work is applicable over the complete range of ionic concentration.

## 2. Governing equations

While the application is for flow in microchannels that have rectangular or trapezoidal cross- section, the present analysis is applied to an infinite parallel plate channel. To apply the governing Navier–Stokes and energy equations to this situation, the following simplifying assumptions are made:

1. The flow is laminar, incompressible, steady, fully developed hydrodynamically and thermally, and the channel is considered as infinite parallel plates.
2. Gravity forces are ignored.
3. The fluid is Newtonian and its properties are independent of the local electric field strength.
4. The ions are point charges, with no concentration gradients in the flow.
5. Zeta potential is assumed to be uniform over the surface.

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6. Viscous dissipation is neglected as its magnitude is very small in microchannel flows and the pressure force is the dominating factor.
7. The fluid is a continuum, and the Knudsen number is <0.1.

Application of these assumptions to the three governing equations results in:
Continuity equation:

$$\frac{\mathrm{d}u}{\mathrm{d}x} = 0 \tag{1}$$

Navier–Stokes equation:

$$\mu \frac{\mathrm{d}^2 u}{\mathrm{d}y^2} - \frac{\mathrm{d}p}{\mathrm{d}x} + \rho_\mathrm{e} E_x = 0 \tag{2}$$

Energy equation:

$$u \frac{\mathrm{d}T}{\mathrm{d}x} = \alpha \left( \frac{\partial^2 T}{\partial y^2} \right) \tag{3}$$

where $x$ is the direction along the flow and $y$ is the direction perpendicular to fluid flow and is measured from the channel centerline. The forces acting on an element of liquid include the pressure force, the viscous force and the electrical body force generated by the flow-induced electro-kinetic field (i.e., the streaming potential) represented by the term $\rho_\mathrm{e} E_x$, where $\rho_\mathrm{e}$ is the net charge density per unit volume and $E_x$ is the non-dimensional streaming electric field. We must develop an expression for $\rho_\mathrm{e} E_x$ before the above equations can be solved.

### 2.1. Development of surface charges and electric double layer

Any surface is likely to carry some charges because of "broken bonds" and "surface charge traps". Likewise, most surfaces acquire an electrostatic charge when in contact with an aqueous solution. If the liquid contains a very small amount of ions (due to impurities), the presence of a surface charge causes both counter-ions and co-ions in the liquid to be preferentially redistributed, leading to the formation of the electric double layer (EDL) near the wall.

The EDL can be divided into an *inner compact layer* and an *outer diffuse layer*. Ions of opposite charges cluster close to the wall, forming the Stern layer or the Shear Plane, and the ions within the Stern layer are attracted to the wall with very strong electrostatic forces. The wall electrostatic attraction causes the counter-ion concentration to be higher near the solid surface as compared to the bulk fluid away from the wall. Contrary to this the co-ion concentration near the surface is lower than that in the bulk liquid, due to the electrostatic repulsion. In contrast, ions in the diffuse layer are less affected by the charged surface (than those in the compact or inner layer) and, hence, are mobile. The thickness of the diffuse layer is dependent on the bulk ionic concentration and electrical properties of the liquid. Electrostatic potential is generally measured at the shear plane, where the electric potential is measurable and is called the *zeta potential*, denoted by $\zeta$, which typically decays exponentially at distances farther from the wall.

The fluid flow in the microchannel results in the downstream flow of the counter-ions. This causes an electric current, called the streaming current, in the direction of fluid flow. The streaming potential associated with the streaming current is called electro-kinetic potential. This potential drives the counter-ions in the direction opposite to the streaming current. When the ions move in the liquid, they exert a force on the liquid molecules, thus generating a viscous effect, usually referred to as the electro-viscous effect. Generally, for macrochannel flow the EDL effects can be safely neglected, as the thickness of the EDL is very small compared with the hydraulic diameter of the channels. However, for microchannel flow the thickness of the EDL is often comparable with the characteristic size of the channels and cannot be neglected.

### 2.2. Poisson–Boltzmann equation

The electrostatic potential $\psi$ is related to the local net charge density per unit volume $\rho_\mathrm{e}$ at certain points in the solution by the Poisson equation as:

$$\frac{\mathrm{d}^2 \psi}{\mathrm{d}y^2} = -\frac{\rho_\mathrm{e}}{\varepsilon} \tag{4}$$

where $\varepsilon$ is the permittivity or the dielectric constant of the solution. Assuming that the equilibrium Boltzmann distribution equation is applicable, the number concentration of the type-$i$ ions, denoted by $n_i$, in a symmetric electrolyte solution is of the form:

$$n_i = n_{\mathrm{io}} \exp\left( -\frac{z_i e \psi}{k_\mathrm{B} T} \right) \tag{5}$$

where $n_{\mathrm{io}}$ and $z_i$ are bulk ionic concentration and the valence of type-$i$ ions, respectively, $e$ is the charge of an electron, $k_\mathrm{B}$ is the Boltzmann constant and $T$ is the absolute temperature.

For a symmetric electrolyte of valence $z$, the net volume charge density $\rho_\mathrm{e}$ is related to the total concentration difference between the cations and anions as:

$$\rho_\mathrm{e} = ze(n_+ - n_-) \tag{6}$$

Substituting the values of the number concentration of each ion from Eq. (5) into Eq. (6), we obtain:

$$\rho_\mathrm{e} = -2zen_\mathrm{o} \sinh\left( \frac{ze\psi}{k_\mathrm{B} T} \right) \tag{7}$$

where $n_\mathrm{o}$ is the bulk ion concentration of each ion.

Substituting the value of charge density (Eq. (7)) in the Poisson equation (Eq. (4)) results in:

$$\frac{\mathrm{d}^2 \psi}{\mathrm{d}y^2} = \frac{2zen_\mathrm{o}}{\varepsilon} \sinh\left( \frac{ze\psi}{k_\mathrm{B} T} \right) \tag{8}$$

The above non-linear second-order one-dimensional equation is known as the Poisson–Boltzmann equation.