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On Taylor dispersion effects for transient solutions in geothermal heating systems

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ABSTRACT

Residential geothermal heating systems have been developed over the past few decades as an alternative to fossil-fuel based heating. Through mathematical modeling the relationship between the operating parameters of the heat pump and the piping length of the geothermal system, which is directly correlated to the cost of the system is investigated. The effect of Taylor dispersion of heat in the fluid which is not yet addressed in the literature with respect to geothermal systems is included. A model of a simple configuration of a single pipe surrounded concentrically by grout and then by soil is considered, where the soil region has a constant ambient temperature. The conduction between the two regions is modeled with a classical thermal resistance. Taylor dispersion effects are significant at higher Peclet numbers associated with this system, and Taylor dispersion in the fluid and thermostat frequency dictate the minimum tubing length needed for successful operation in an insulated subsystem. We consider both steady state and transient (cyclic operation) analyses and find that the axial dispersion increases linearly in the cycle rate for large flow rates. We find that the estimated tubing length for complete energy transport is increased when Taylor dispersion is included, but that this effect can be mitigated with an appropriate choice of the borehole radius.

1. Introduction

Although the promise of environmentally friendly, low-cost energy harnessing for heating and cooling of residential properties has been known for nearly 30 years, the adoption of the technology in the United States has been slow. These geothermal systems, also known as ground-coupled heat pumps, consist of a field of vertical boreholes in the ground with pipes carrying a heat transfer fluid into the earth to gain access to the stable year-round temperatures underground. The fluid is pumped back to the residential unit to be used for heating or cooling depending on the season. A significant portion of the cost for ground-coupled heat pump systems is in the installation of the large networks of piping to harness the geothermal energy. These installation costs are currently cost-prohibitive, with a typical return-time on investment on the order of 8–10 years. One means to improve the economic competitiveness of these systems is to reduce the installation footprint. Our focus in this research program is to develop mathematical models to quantify how the length of the piping is related to the operational parameters of the system. The model developed in this work includes both the effect of cycling (turning the fluid flow on or off in response to the heating or cooling load of the residence) and the effect of axial heat transport, by means of advection and Taylor dispersion, in the pipes.

The main design criteria for these heating systems is the effective power that can be obtained from the fluid heated as it flows through the tubing.[2] The power rating of these systems can be estimated by determining the change in rate of thermal energy of the fluid entering the system from the residence and leaving the system

$$\text{Power} = \rho_w c_w U^* A_p \Delta T,$$

where ρ_w is the density of the fluid, c_w is the specific heat of the fluid, U^* is the characteristic fluid velocity, A_p is the pipe's cross-sectional area, and ΔT is the temperature change. For a given fluid, such as water or ethylene glycol, flow rate and power rating requirement, the required length of pipe needed for the system to function properly is determined from the unknown temperature variation in the axial direction. However, the temperature profile in the fluid is necessarily coupled to the thermal behavior in the soil from which the energy is transferred. In order to fully understand how these systems work, a requirement for design optimization, the temperature profile in both the soil and the fluid need to be solved simultaneously. This is a difficult modeling task, so it is no surprise that some simplifications in the modeling have been attempted in order to understand different aspects of the system.

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[2] We focus in this work on closed-loop systems.

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Nomenclature

D	dimensionless axial diffusion coefficient
H	dimensionless heat transfer coefficient
L	dimensional characteristic axial length
Nu	scaled Nusselt number, $L^2h/(ak_g)$
P	period of thermostat oscillation (on and off cycles)
Pe	Peclet number, U^*a/α_w
R	radial extent of grout region
T	temperature
U	characteristic axial fluid velocity
a	tubing radius
c	specific heat (J/kg K)
h	heat transfer coefficient
k	thermal conductivity
ℓ	dimensionless axial characteristic length scale
r	radial coordinate
t	time
u	axial fluid velocity
x	axial coordinate
v	thermal front velocity
y	moving frame of reference, $x - vPet$

Greek symbols

α	thermal diffusivity
δ	relative temporal period of oscillation compared to characteristic time-scale
ϵ	aspect ratio, a/L
κ	wavenumber of axial temperature profile
η	dimensionless thermostat cycle rate
ρ	density
σ	spatial exponential growth rate (steady-state solutions)
τ	slow-time, ϵt
θ	dominant grout temperature
$\bar{\theta}$	correction to radial average temperature
ξ	similarity variable, $y/2\sqrt{Dt}$

Subscripts

F	Fourier-law diffusion
T_w	Taylor dispersion effect from water
T_g	Taylor dispersion effect from grout layer
$0, 1, \ldots$	correction of quantity to $O(\epsilon^{0,1,\ldots})$
a	ambient
eff	effective quantity (time-averaged)
g	grout quantity
i	inlet
r	$\partial/\partial r$
s	steady
t	$\partial/\partial t$
u	unsteady
w	water quantity
x	$\partial/\partial x$
τ	$\partial/\partial \tau$

Superscripts

$*$	dimensional quantity
(1)	water quantity
(2)	grout quantity

Analytical approaches to these systems have focused on the thermal behavior of the soil in the cross-section, with the assumption that the temperature profile in the fluid is known. The simplest model used is an adaption of the Kelvin line-source model [1]. This model assumes that a radial heat flux is known from the tubing which is proportional to the temperature difference between the fluid temperature and the soil temperature. To model axial heat transport, the cylinder source model has been applied [2] (a brief and clear review of this models is presented in [3]). This model couples the heat flow between cross-sectional planes of the line-source model with a prescribed thermal resistance [4–6]. Further, a radial thermal resistance is used to decouple the local thermal behavior from the far-field behavior. All of these attempts have not investigated how the advective heat transport in the fluid affects the axial heat flow in the soil in a direct, physically fundamental way. A fundamental approach does not rely on a phenomenologically-based choice for an axial thermal resistance, which then must be modified with a new series of experiments for each new system.

There has been recent interest [3] in developing transient models that can provide better analyses of the short time behavior of the ground heat exchanger (GHE). Dobson [7] showed that cycling the flow (with an on-time on the order of minutes) can improve the efficiency of the system. The development of better transient models will enable better simulation of geothermal systems and improve the optimization of geothermal designs. We are interested in finding a mathematical description, based on the fundamental equations of heat transfer in continuous media, of the near and far-field behavior of the system in order to optimize their design. In this work, we consider the local behavior near the tubing, and include the effect of Taylor dispersion of heat in the tubing and the grout for the simple case of a single pipe within a borehole. Although we assume knowledge of the far-field temperature profile in this work, a subsequent paper in preparation addresses how this local temperature profile is coupled to the far-field distribution over long times.

In order to better understand the dominant mechanisms of the local system, we note that there are two time-scales of interest. The first corresponds to the thermal transport time due to conduction for heat to diffuse through soil, which is on the order of hours. The second time scale is the typical cycle time needed to maintain a residence at a prescribed temperature, which is on the order of minutes. These time scales can be represented mathematically by

$$\text{conduction time} = \frac{a^2}{\alpha_g}, \quad \text{cycle time} = \frac{a}{U^*},$$

where a is the radius of the pipe and α_g is the thermal diffusivity of the grout. The ratio of these time-scales

$$\frac{\text{cycle time}}{\text{conduction time}} = \frac{a\alpha_g}{a^2U^*} = \frac{\alpha}{Pe} \ll 1,$$

where $\alpha = \alpha_g/\alpha_w$ is the ratio of the thermal diffusivities of the grout to the water and $Pe = aU^*/\alpha_w$ is the Peclet number of the fluid flow. Since $\alpha = O(1)$, this suggests that we are interested in the case for large Peclet numbers.

There is a classical result from solutal diffusion in laminar fluid flows found by Taylor [8,9], in which he found an effective diffusion coefficient for the concentration C in a solvent

$$\frac{\partial C}{\partial t} = \left[1 + \frac{Pe^2}{192}\right]\frac{\partial^2 C}{\partial y^2},$$

where time t is on the diffusive time-scale and y is a frame of reference moving with the average fluid velocity. The first term in the effective diffusion coefficient represents Fickian diffusion, whose relative importance decreases with increasing Peclet number. The second term, however, grows quadratically with increasing Peclet number, and this term is called Taylor dispersion. Further, Aris