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Unsteady boundary layer flow and heat transfer past a porous stretching sheet in presence of variable viscosity and thermal diffusivity

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ABSTRACT

The aim of this paper is to present the unsteady boundary layer flow and heat transfer of a fluid towards a porous stretching sheet. Fluid viscosity and thermal diffusivity are assumed to vary as linear functions of temperature. Using similarity solutions partial differential equations corresponding to the momentum and energy equations are converted into highly non-linear ordinary differential equations. Numerical solutions of these equations are obtained with the help of shooting method. It is noted that due to increase in unsteadiness parameter, fluid velocity decreases up to the crossing over point and after this point opposite behaviour is noted. The temperature decreases significantly in this case. Fluid velocity decreases with increasing temperature-dependent fluid viscosity parameter (i.e. with decreasing viscosity) up to the crossing over point but increases after that point and the temperature decreases in this case. Due to increase in thermal diffusivity parameter, temperature is found to increase.

1. Introduction

The study of hydrodynamic flow and heat transfer over a stretching sheet has gained considerable attention due to its applications in industries and important bearings on several technological processes. Crane [1] investigated the flow caused by the stretching of a sheet. Many researchers such as Gupta and Gupta [2], Chen and Char [3], Dutta et al. [4] extended the work of Crane [1] by including the effect of heat and mass transfer analysis under different physical situations.

All the above mentioned studies confined their discussions by assuming uniformity of fluid viscosity. However, it is known that the physical properties of fluid may change significantly with temperature. The increase of temperature leads to a local increase in the transport phenomena by reducing the viscosity across the momentum boundary layer and so rate of heat transfer at the wall is also affected. Therefore, to predict the flow behaviour accurately, it is necessary to take into account the viscosity variation for incompressible fluids.

Gary et al. [5] and Mehta and Sood [6] showed that, when this effect is included the flow characteristics may change substantially compared to constant viscosity assumption. Recently Mukhopadhyay et al. [7] investigated the MHD boundary layer flow with variable fluid viscosity over a heated stretching sheet.

All of the above mentioned studies were restricted to the steady state conditions. The transient or unsteady aspects become interesting in certain practical problems where the motion of the stretched surface may start impulsively from rest. Elbashbeshy and Bazid [8] and Sharidan et al. [9] presented similarity solutions for unsteady flow and heat transfer over a stretching surface.

The present work deals with unsteady fluid flow and heat transfer over a stretching sheet in presence of wall suction. Fluid viscosity and thermal diffusivity are assumed to vary as linear functions of temperature. Similarity variable and similarity solutions are obtained and using them, a third order and a second order ordinary differential equations corresponding to momentum and energy equations are derived. These equations are solved numerically using shooting method. The effects of different parameters (viz. unsteadiness, temperature-dependent fluid viscosity, variable thermal diffusivity and suction) on velocity and temperature fields are investigated and analysed with the help of their graphical representations.

2. Equations of motion

We consider unsteady two-dimensional forced convection flow of a viscous incompressible fluid past a heated stretching sheet immersed in a porous medium in the region $y > 0$ and moving with non-uniform velocity $U(x,t) = \frac{cx}{1-\alpha t}$ where c, α are positive constants with dimensions $(\text{time})^{-1}$, c is the initial stretching rate and $\frac{c}{1-\alpha t}$ is the effective stretching rate which is increasing with time. In case of polymer extrusion, the material properties of the extruded sheet may vary with time. Here, the stretching surface is subjected to such amount of tension which does not alter the structure of the porous material.

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Nomenclature

A	fluid viscosity variation parameter
f	non-dimensional stream function
f'	first order derivative with respect to η
f''	second order derivative with respect to η
f'''	third order derivative with respect to η
M	unsteadiness parameter
Pr	Prandtl number
p,q	variables
S	suction parameter
T	temperature of the fluid
T_w	temperature of the wall of the surface
T_∞	free-stream temperature
u, v	components of velocity in x and y directions
z	variable
Greek symbols	
β	thermal diffusivity parameter
η	similarity variable
k	the non-uniform value of coefficient of thermal diffusivity
μ	dynamic viscosity
μ^*	reference viscosity
v^*	reference kinematic viscosity
ψ	stream function
ρ	density of the fluid
θ	non-dimensional temperature
θ'	first order derivative with respect to η
θ''	second order derivative with respect to η

The temperature of the sheet is different from that of the ambient medium. The fluid viscosity is assumed to vary with temperature while the other fluid properties are assumed constants.

The continuity, momentum and energy equations governing such type of flow are written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)=\frac{1}{\rho}\frac{\partial \mu}{\partial T}\frac{\partial T}{\partial y}\frac{\partial u}{\partial y}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\kappa\frac{\partial T}{\partial y}\right), \tag{3}$$

where u and v are the components of velocity, respectively, in x and y directions, T is the temperature, κ is the coefficient of thermal diffusivity (dependent on temperature), c_p is the specific heat, ρ is the fluid density (assumed constant), μ is the coefficient of fluid viscosity (dependent on temperature), k is the permeability of the porous medium.

2.1. Boundary conditions

The appropriate boundary conditions for the problem are given by

$$u=U(x,t),\quad v=v_w(t),\quad T=T_w(x,t)\qquad \text{at } y=0, \tag{4}$$

$$u\to 0,\quad T\to T_\infty \qquad \text{as } y\to\infty, \tag{5}$$

where $v_w(t)=-v_0\sqrt{\frac{1}{1-\alpha t}}$ is the velocity of suction ($v_0>0$) of the fluid, $T_w(x,t)=T_\infty+\frac{1}{2}T_0\mathrm{Re}_x(1-\alpha t)^{\frac{-1}{2}}$ is the wall temperature [10] where $\mathrm{Re}_x=\frac{Ux}{v^*}$ is the local Reynolds number based on the stretching velocity U, T_0 is a reference temperature such that $0\leqslant T_0\leqslant T_w$ and v^* is the kinematic viscosity of the ambient fluid. The expressions for $U(x,t)$, $T_w(x,t)$, $v_w(t)$ are valid only for time $t<\alpha^{-1}$ unless $\alpha=0$.

It is to be noted that though the velocity and temperature are time dependent (initially), no initial condition is needed in the boundary as the transformed equations [see (9) and (10)] and the boundary conditions [see (11) and (12)] are independent of "t" (see Elbashbeshy and Bazid [8], Andersson et al. [10]). On the other hand if the initial and boundary conditions are taken as [instead of (4) and (5)]

$$t<0:\quad u=0,\quad T=T_\infty \qquad \text{for any } x,y, \tag{4a}$$

$$t\geqslant 0:\quad u=U(x,t),\quad v=v_w(t),\quad T=T_w(x,t)\qquad \text{at } y=0, \tag{4b}$$

$$u\to 0,\quad T\to T_\infty \qquad \text{as } y\to\infty \tag{5}$$

then also these conditions (4) and (5) reduce to Eqs. (11) and (12).

2.2. Method of solution

We now introduce the following relations for u, v and θ as

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x}\quad\text{and}\quad \theta=\frac{T-T_\infty}{T_w-T_\infty}, \tag{6}$$

where ψ is the stream function.

The temperature-dependent fluid viscosity is given by [7]

$$\mu=\mu^*[a+b(T_w-T)], \tag{7}$$

where μ^* is the constant value of the coefficient of viscosity far away from the sheet and a, b are constants and $b(>0)$.

We have used viscosity–temperature relation $\mu=a-bT$ $(b>0)$ which is in perfect harmony with the relation $\mu=e^{-aT}$ [11] when second and higher order terms neglected in the expansions.

The variation of thermal diffusivity with the dimensionless temperature is written as

$$\kappa=\kappa_0(1+\beta\theta). \tag{8}$$

β is a parameter which depends on the nature of the fluid, κ_0 is the value of thermal diffusivity at the temperature T_w.

We introduce

$$\eta=\sqrt{\frac{c}{v^*(1-\alpha t)}}y,\quad \psi=\sqrt{\frac{v^*c}{(1-\alpha t)}}xf(\eta),$$

$$T=T_\infty+T_0\left[\frac{cx^2}{2v^*}\right](1-\alpha t)^{\frac{-3}{2}}\theta(\eta).$$

With the help of the above relations, the governing equations finally reduce to

$$M\left(\frac{\eta}{2}f''+f'\right)+f'^2-ff''=-A\theta'f''+(a+A)f'''-A\theta f''', \tag{9}$$

$$\frac{M}{2}\eta\theta'+\frac{3}{2}M\theta+2f'\theta-f\theta'=\frac{1}{\mathrm{Pr}}(\beta\theta'^2+\theta''+\beta\theta\theta''), \tag{10}$$

where $M=\frac{\alpha}{c}$ is the unsteadiness parameter, $A=b(T_w-T_\infty)$ is the temperature-dependent viscosity parameter, $v^*=\frac{\mu^*}{\rho}$.

The boundary conditions (4) and (5) then become

$$f'=1,\quad f=S,\quad \theta=1\qquad \text{at } \eta=0, \tag{11}$$

$$\text{and } f'\to 0,\quad \theta\to 0\qquad \text{as } \eta\to\infty, \tag{12}$$

where $\mathrm{Pr}=\frac{v^*\rho c_p}{\kappa}=\frac{\mu^* c_p}{\kappa}$ is the Prandtl number, $S=\frac{v_0}{\sqrt{v^*c}}$, $S>0$ corresponds to suction.